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Number Sense

STATE GOAL 6:

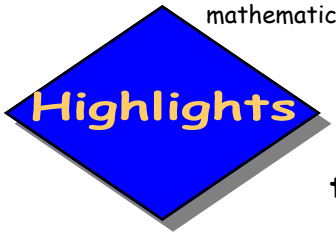
Demonstrate knowledge and use of numbers and their representations in a broad range of theoretical and practical settings.

Statement of Purpose

The ability to communicate with numbers is necessary in many areas of our lives. We need to be able to speak, read and write about numerical information. During the late elementary years students develop number sense as they gain experience with small and large numbers. They use familiar quantities to compare and make sense out of unfamiliar quantities. There are four aspects of number sense which form the foundation of this module and promote the ability to reason quantitatively. These four aspects are:

- ***Quantities***—working and communicating comfortably with quantities that represent counts, measurements, and dates;
- ***Representations***—understanding that a quantity can be represented by many different equivalent notations. These often require different labels and may best be represented by different types of graphs, tables, or expressions;
- ***Relationships*** between quantities—being able to make comparisons, and expressing these comparisons as differences or ratios ;
- ***Patterns***—recognizing patterns, and representing these patterns can help us to discover trends in numerical data, and help us reason inductively.

Number sense is an important component of many content areas and processes of mathematics, including algebra, geometry, probability/statistics, and measurement. Increased number sense also enhances our ability to reason mathematically and to gain a greater understanding of quantitative relationships in all situations.



In this unit, we will explore the four aspects of number sense listed above. The unifying activity of the module is to keep a journal that reflects those understandings through written communication and illustrations of numerical facts.

Connections to the Illinois Learning Standards.

Standard 6.A.—Demonstrate knowledge and use of numbers and their representations in a broad range of theoretical and practical settings. Representations of fractional parts as common fractions, decimals fractions and percents is emphasized throughout. One activity looks at numerical information related to the Statue of Liberty. The "Who Are We" poster project asks students to represent numerical information about their group.

Standard 6.B.—Investigate, represent and solve problems using number facts, operations (addition, subtraction, multiplication, division) and their properties, algorithms and relationships. Activities such as the Statue of Liberty Proportions address this standard.

Standard 6.C.—Compute and estimate using mental mathematics, paper-and-pencil method, calculators and computers. Estimation is key to the Statue of Liberty activity.

Standard 6.D.—Solve problems using comparison of quantities, ratios, proportions and percents. Every activity in this module is designed to require students to make these comparisons.

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Table of Contents

Note: Appendices are printed only on the odd pages. This is done to make photocopying easier. That is, each participant should have a copy of all the odd numbered pages. While the instructors should have a copy of all the pages.

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Materials

MATERIALS LIST

- Calculators
- **Poster board**
- **Reference materials for U. S. history**
- **Spreadsheet**
- **Construction paper**
- **100 counters**
- **Journals**
- **100 pennies (minimum)**
- **Rubber stamp for hundred grid**
- **Eyedroppers**
- **Internet access**
- **Scissors**
- **Colored markers**
- **Rectangular prisms for the water flask problem**

Opening Activity: The Statue of Liberty

Instructor Page

A primary goal of these number sense activities is to build an understanding of how numbers are used in many different aspects of life.

Students use familiar quantities and make comparisons to make sense out of less familiar quantities.

Students need to talk about their thinking. The discussion following this activity is important. Students need to share their thinking strategies with the group.

Context:

This passage describes the Statue of Liberty. Several numerical facts are stated, except that the numbers are removed from the statements. Students must estimate reasonable counts, dates, and measurements, then choose which number (from the list of numbers on the right) belongs in each blank. Some use of measurement equivalents is required in order to be able to choose the correct number.

Activity Instructions:

⇒ Distribute copies of the Statue of Liberty activity to all students. Ask the students work independently for 10 – 15 minutes filling in the blanks with the numbers. Remind students that the numbers are grouped for each paragraph.

⇒ Arrange students in groups of two or three to compare and discuss their answers. Ask them to explain their reasoning or show their calculations where appropriate.

⇒ When most groups have finished, ask some groups to report their results and procedures for verifying their answers. Ask students to tell which number in a section was easiest to place in the text. Did other students think about that section differently?

⇒ Ask students to choose one place where they did not write the correct answer the first time. What was their thinking when they first chose the number? What made them change it?

Journal Writing:

Write another numerical sentence describing the Statue of Liberty. Write a comparison sentence to make your number fact easier to understand.

Discussion of Math Content and Related Questions:

- Discuss the different ways that we use numbers in everyday life. Can you separate the different numbers in this article into different categories? Which ones? Why did you decide to categorize the numbers in this particular way?
- For example, sometimes numbers are used as ***counts***—there are 354 steps, the seven rays of the crown. Other times, we use numbers as ***measures***—the Statue weighs 225 tons and is 306 feet tall. Also, some numbers are ***dates*** or measures of time.
- Often, we use familiar quantities as benchmarks to make comparisons that help us make sense of numerical information—the length of the Statue of Liberty's nose is about the same as a student's height. The use of this kind of comparison to give "reality" to quantities leads to the idea of ***ratios***.
- Another important aspect of this activity and discussion are the use of labels: What labels are appropriate for a particular use of a number as a count? As a measure? As a ratio?

- Hawkes, Nigel. *Structures*. Macmillan Publishing Company, New York,

NOTE: Solutions for this activity are in the appendix to this module.

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The Statue of Liberty

In the following paragraphs about the Statue of Liberty, fill in the blanks with the appropriate quantities that complete the information. The numbers to be used in the paragraph are located in the list to the right:

The Statue of Liberty stands on Bedloe's Island in Upper New York Bay. She holds her lighted torch high above her head and has been a symbol of freedom and a promise of new life since _____ .

In _____ Frederic Auguste Bartholdi announced that he wanted the people of France to give a gift to the United States to celebrate their centennial (_____ year) anniversary of the Declaration of Independence. This gift would become the Statue of Liberty. Bartholdi had actually started building the Statue in _____ . He finished it _____ years later in July, _____. It took him about the same number of years to build the statue as the average fourth-grade student has been alive.

9
100
1875
1876
1884
1886

The Statue of Liberty was a joint effort between the United States and France. The American people were to build the pedestal and the French people were responsible for the Statue. In both countries the money was raised by donations.

After it was finished the Statue made the ocean journey from France to the U. S. For the trip the Statue was in _____ pieces and packed in _____ crates. It took _____ months to put the Statue back together on its new pedestal. On October 28, _____, the dedication of the Statue of Liberty took place in front of thousands of spectators. She was a centennial gift _____ years late.

four
ten
214
350
1886

The Statue was constructed of _____ sheets of thin (_____ inch) beaten copper that were hung on an iron frame. Over the years the weather caused the iron to react with the copper and caused corrosion. In the _____'s the iron armature rods were replaced with stainless steel rods. The workers removed a few of the old rods at a time and replaced them with exact copies. It took a year to replace _____ feet of rods. If these rods were laid end to end, they would be almost as tall as _____ Sears Towers.

3/32
7
300
1980
10,000

At the time she was built, the Statue was the largest in the world. The height from the base of the statue to the torch is _____ feet and from the ground beneath the pedestal to the tip of the torch is _____feet. The length of her hand is _____ feet, and the length of her index finger is _____ feet. The distance across one of her eyes is _____ feet, while the length of her nose is _____ feet. That's about the same as the height of an average fifth grade student. The total weight of the copper in the Statue is _____ pounds (____ tons), and the weight of the steel is _____ pounds (_____ tons). Her total weight, including all parts of the structure (copper, steel, glass, etc.), is _____ tons.

2.5
4.5
8
$16\frac{5}{8}$
31
151
225
306
62,000
125
250,000

Visitors climb _____ steps to reach the crown or _____ steps in order to reach the top of the pedestal. In most houses there are _____ steps between floors. That means the climb to the top of the pedestal is the same as _____ floors.

There are _____ windows in the crown. They symbolize gemstones found on earth. The _____ rays of the Statue's crown represent the _____ seas and _____ continents.

seven
seven
seven
twelve
sixteen
25
192
354

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The classroom chart is a large piece of paper divided into 100 squares. A penny is attached to a square every day. Students also have a copy of the 100 square in their journals. They mark off a square daily and record in their journals daily as another student records on an overhead 100 square. The goal is to reach $1.00 or one whole.

We are using an area model to show hundredths. The shape of the fractional part can change, but the area stays the same.

Day 1 Journal
Penny Chart Activity

Activity Instructions:

Step One:
Students should be able to suggest that each penny can be recorded four ways: as money, a decimal, a percent, and a fraction because a penny is .01 of a dollar, 1% of a dollar, and 1/100 of a dollar.
For example, when the first penny is placed on the chart record:
$.01. .01, 1%, 1/100
When the 79th penny is placed on the chart record:
$.79, .79, 79%. 79/100

Step Two: Lowest-term fractions.
On the 5th day record:
$.05. .05, 5%, 5/100 or 1/20
Show that 5/100 is equivalent to 1/20 by having 20 strips of paper each the size of 5 squares. Cover the 100 square. One strip is 1/20 of all the strips needed to cover the 100 square. Remind students that five pennies is equal to one nickel and 20 nickels is the same as one dollar - 5 cents = 1/20 of a dollar.

Look back at the previous four days to see if some of these can be expressed as lowest-term fractions. Use the strips of paper method. (2/100 = 1/50, 4/100 = 1/25)

On the 10th day record:
$.10, .10 or .1, 10%, 10/100 or 1/10
Show that 10/100 is equivalent to 1/10 by having 10 strips of paper each the size of 10 squares. Cover the 100 square. One strip is 1/10 of all the strips needed to cover the 100 square.

Discussion of Math Content and Related Questions:

- Each day the students are using something familiar, the money, to expand to less familiar ideas of decimal fractions, percents and common fractions.
- They are recording several equivalent forms for the same value.
- Both the coins and the hundredths chart are symbolic representations of the idea of fractional parts.

Resource Connections:

About Teaching Mathematics by Marilyn Burns
50 Problem-Solving Lessons by Marilyn Burns

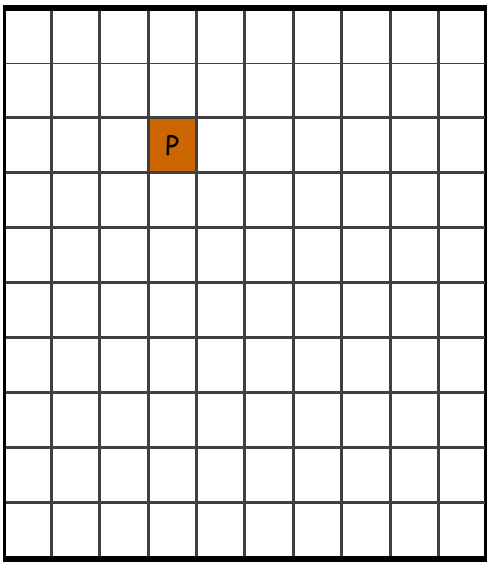
Literature Connections:

100 Penny Box by Sharon Bell Mathis
100 Dresses by Eleanor Estes
Emily's First 100 Days of School by Rosemary Wells
The $1.00 Word Riddle Book by Marilyn Burns

Participant Page

Day 1 Journal
Penny Chart Activity

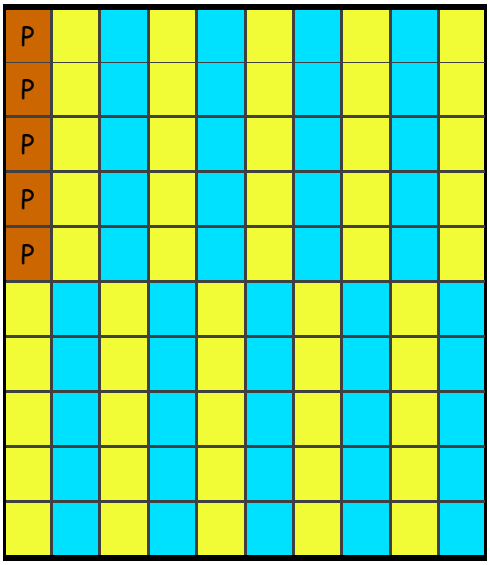
- *Each day add another penny to the chart and shade in an additional square in your journal. Record the value of the pennies in four ways.*
- *Everyone does not have to shade the same squares of their hundred grid; just the same number of squares. The fractional parts are still the same.*
- *On day four or day five, explore the different ways can you shade four or five*



On the **first day** shade in one square of the hundred chart for one penny. Record the value of one penny in four ways: as money, a decimal, a percent, and a fraction because a penny is .01 of a dollar, 1% of a dollar, and 1/100 of a dollar.

When the first penny is placed on the chart, record:

Money	$.01
Decimal fraction	.01
Percent	1%
Common fraction	1/100



On the **fifth day** shade in five squares of the hundreds chart for five pennies. Record the value of the pennies in four ways. Show that 20 strips of five cover all 100 squares so 5/100 = 1/20.

When the fifth penny is placed on the chart, record:

Money	$.05
Decimal fraction	.05
Percent	5%
Common fraction	5/100
Lowest-term fraction	1/20

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Opening Activity—100 Things

Each pair of students counts out exactly 100 of their "things". For example, there might be 100 buttons, 100 rocks, or 100 birthday candles. It is important to be exact. Remind everyone that percent means "out of 100". Then ask each group to hold up 2% of their things. Then hold up 3% of their things, 4% of their things, 12%, 25%, 50%, 80%. Students quickly realize it is easy to find the percent when you have exactly 100 things.

Next, ask the student pairs to separate their "100 things" into smaller groups based on characteristics such as size, color, or shape. Each pair should write the percent that is in each of their smaller groups and find the total of the percents. Since the smaller groups are parts of the whole 100, students should find that their sum must be 100%.

Create a Mosaic

Context: Mosaics are forms of art that have been used for centuries. They use small colored pieces arranged into a pleasing pattern, design, or picture.

Activity Instructions:

⇒ Students use construction paper to make the pieces of a mosaic.

⇒ The mosaic uses exactly 100 pieces of paper. The size and shape of the pieces are not important (we are focusing on the number of pieces), but there must be exactly 100 pieces because the focus is on percents, and percent means "out of 100".

⇒ When students have completed the mosaic, they count the number of each color used, and then complete the chart.

Extensions:

- Search the Internet for exactly 100 examples of mosaics. Make a chart that lists the artist and the country the artist is from. Determine the percent of times each country is mentioned on your list.
- Think of a survey question. Survey 100 people. Report your result to the class using percents and fractions.
- Open a book to any page. Count the first 100 letters shown on the page. Make a tally showing the number of times each letter of the alphabet is used in the first 100 letters of the page. Report your result in percents and fractions. Compare your results to others in the classroom. Which letters are used most often? Based on the information gathered from the class, how many times would you expect the letter "e" to appear in the first 100 letters of Lincoln's Gettysburg Address? the first 200 letters? the first 300 letters? Get a copy of the Address and check.

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Your mosaic must be composed of exactly 100 pieces of paper. The pieces can be any size or shape.

Create a Mosaic

- Mosaics are forms of art that have been used for centuries. They use small colored pieces arranged into a pleasing pattern, design, or picture.
- Use construction paper to make the pieces of your mosaic.
- Use exactly 100 pieces of paper in your mosaic. The size and shape of your pieces are not important, but you must use exactly 100 pieces because we are focusing on percents, and percent means "out of 100".
- When you have completed your mosaic, count the number of each color you used. Complete and attach this chart to your mosaic.

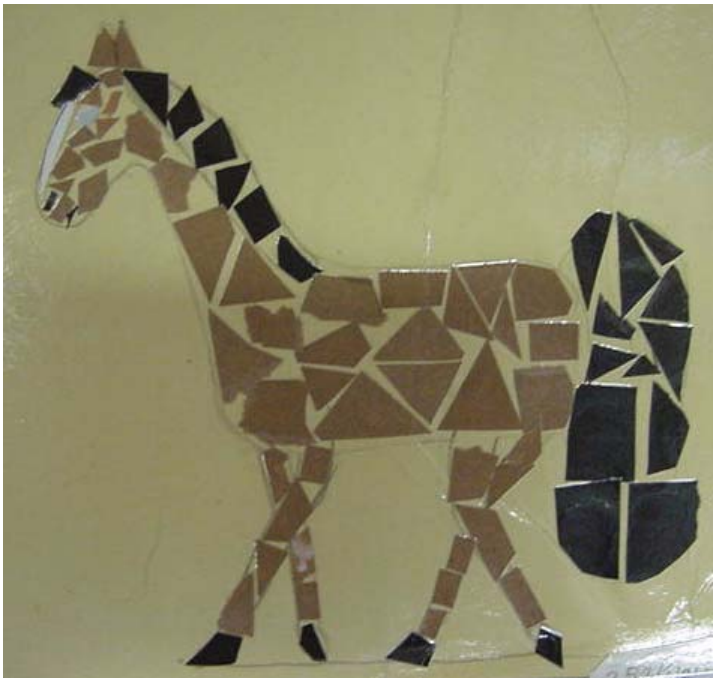
Color	Number of pieces of this color	Fraction of pieces of this color	Percent of pieces of this color

Check:

1. Is the total number of pieces 100?
_____

2. Do the percents add to 100%? _____
Why should they?
_____
_____
_____

3. Do the fractions add to one whole?
_____
Why should they?
_____
_____
_____



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Skip Charts to Solve Proportions

Instructor Page

Eventually, students will want to know how to calculate the percent of a group if they do not have exactly 100 things in the group. They now need to be introduced to proportions. Proportions help us compare groups of different sizes

- First, use counters and a "skip chart" to show and solve proportions.

triangles	Δ Δ Δ	Δ Δ Δ Δ Δ Δ	Δ Δ Δ Δ Δ Δ Δ Δ Δ	Δ Δ Δ Δ Δ Δ Δ Δ Δ Δ Δ Δ	Δ Δ Δ Δ Δ Δ Δ Δ Δ Δ Δ Δ Δ Δ Δ
quadrilaterals	◊ ◊ ◊ ◊	◊ ◊ ◊ ◊ ◊ ◊ ◊ ◊	◊ ◊ ◊ ◊ ◊ ◊ ◊ ◊ ◊ ◊ ◊ ◊	◊ ◊ ◊ ◊ ◊ ◊ ◊ ◊ ◊ ◊ ◊ ◊ ◊ ◊ ◊ ◊	◊ ◊ ◊ ◊ ◊ ◊ ◊ ◊ ◊ ◊ ◊ ◊ ◊ ◊ ◊ ◊ ◊ ◊ ◊ ◊

While showing the chart with counters also show how to record with numbers on a chart.

triangles	3	6	9	12	15	18	21
quadrilaterals	4	8	12	16	20	24	28

- After doing several of these examples, students will notice that for each row of the "skip chart" you are just skip counting by multiples of the beginning number.
- Fractions are equivalent if they are in the same "skip chart". For example, all the fractions in this "skip chart" are equivalent. 3/4, 6/8, 12/16, and 18/24 are all equivalent because they are all in the same "skip chart". However, 3/4 and 6/12 are not equivalent because they would not be in the same "skip chart".
- Next, use "skip charts" to solve proportion problems.

There are three dogs for every 5 students. How many dogs would there be if there were 15 students?

dogs	3	6	9	12
students	5	10	15	20

Because 9 corresponds to 15 in the "skip chart", there would be 9 dogs for 15 students.

Allow time for students to solve and write several of these types of problems. As they grow tired of making "skip charts" to answer these problems, encourage students to notice that you multiply by the same number on the top row as you do on the bottom row. Therefore, to get to 9/15 you multiply 3 x 3 to get to 9, because you multiplied 5 x 3 to get to 15.

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Skip Charts to Solve Proportions

Journal Activity
How can you skip directly between two numbers without making the whole chart? Write what you do and tell why it works.

Complete the following skip charts to solve the problems.

Bubble gum	2	4	6				
Cost in cents	5	10					

How much bubble gum can you buy for 30 cents? _____

What is the cost of 14 pieces of bubble gum? _____

Pencils	2	4	6						
Cost in cents	15		45						

What is the cost of 8 pencils? _____

How many pencils can you buy for 30 cents? _____

How many pencils can you buy for $1.50? _____

Donuts	3	6	9		15				
Cost in dollars		2.10							

What is the cost of 3 donuts? _____

How many donuts can you buy for $4.20? _____

How many donuts can you buy for $10.50? _____

What is the cost of one donut? _____

players	9	18							
teams	1								

How many players are needed for 5 teams? _____

How many teams can be made from 100 players? _____

How many players are needed for 12 teams? _____

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Skip Charts to Solve Proportions (continued)

Instructor Page

Allow lots of time for students to write and solve proportion problems that they create. Encourage them to label each problem to be sure they keep dogs with dogs and students with students.

Soon, student will begin to write the skip chart like this, leaving the between numbers blank. They will determine how to multiply the bottom number to get to the new number, and multiply the top number by the same number.

dogs	3		9
students	5		15

5 x 3 = 15
so 3 x 3 = 9

Students will soon solve proportion problems without the skip chart. They will write:

$$\text{Dogs} \quad \frac{3}{5} = \frac{N}{15}$$
$$\text{Students}$$

- Percent problems can be solved in the same way. Use this knowledge to solve percent problems when the total number in the group is not 100. Start by using group totals that are factors of 100. Students can use the "skip chart" strategy to solve percent problems. For example, 3 out of 25 students like to eat green beans. What percent (How many students out of 100?) of the students like to eat green beans?

$$\frac{\text{Students that like green beans}}{\text{Total number of students}} \quad \frac{3}{25} = \frac{N}{100}$$

Students look for some number out of 100 because percent means "out of 100". Because 25 x 4 = 100, they know they should multiply 3 x 4 to get their answer. They also know that 12/100 is 12% from the daily work with the penny chart. So the answer is 12% of the students like green beans. Again encourage students to label their work.

As before, allow time for students to write and solve percent problems of their own. They should use groups that are factors of 100. These would be totals of 2, 4, 5, 10, 20, 25, or 50 because they can use their "skip chart" strategy to solve these. Students soon realize that "skipping forward" involves multiplying and "skipping back" involves dividing. This realization could be used to develop a strategy that works for finding percent for any number. For example, 5 out of 11 students are wearing white shoes. What percent of the students are wearing white shoes?

$$\frac{\text{Students wearing white shoes}}{\text{Total number of students}} \quad \frac{5}{11} = \frac{N}{100}$$

Instead of looking for a number to multiply by 11 to get 100, students can divide 100 by 11 and then multiply the quotient by 5 to get the percent. 100 divided by 11 = 9.0909…, and 9.0909 times 5 = 45.4545

NOTE: Organizers that may help students to create their own proportions are in the appendix to this module.

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More Skip Charts to Solve Proportions

Solve these proportions. Try to find a shortcut.

1. There are 5 players on a basketball team. How many teams can be made from 20 players? ____

players			
teams			

2. We need 3 cups of flour to make 4 dozen cookies. How much flour for 12 dozen? _____

3. His heart beats 12 times in 10 seconds. How many times does it beat in 30 seconds? _____

How many beats in one minute? _____

4. She made 12 free throw out of 20 tries. What percent did she make? _____

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At the beginning of each day of the week group members should meet and add a few facts to the chart. As the week progresses, the group should be identifying the benchmark percents. The information from the table should be illustrated and presented on a poster to the whole class.

Create a "Who Are We?" Poster

Context: Comparisons of part of a group to the whole group using common fractions, decimal fractions and percents.

Activity Instructions:

The goal is to make a poster describing members of your group as part of the whole group using fractions and percents.

Take some time to find out any interesting or trivial facts about your group that can be described numerically. For example: What fractional part of your group has brown eyes? What fractional part of your group has painted fingernails? What fractional part of your group has visited another country? Find out almost anything about your group. Be creative.

Step One:

- Make a list of all the information you discovered that describes your group.

Fact	Part of group	Total number in group	Fraction	Decimal	Percent
Example: Group members that can swim	5	8	5/8	0.625	63%

- Organize your information into a table like this one.
  (To find the decimal, divide 5 by 8 = 0.625 which is the part divided by the total)
  (To find the percent, round the decimal to the nearest one-hundredth, because percent means "out of 100")

Step Two:

- Look for "benchmark" percents.
- Keep surveying your group until you find facts that are close to "benchmark" percents of 0%, 10%, 20%, 25%, 50%, 75%, 100%

  For example:
  0% of our group have walked on the moon.
  10% of our group have an older brother.
  20% of our group have hair less that 2 cm. long.
  25% of our group have a cell phone
  50% of our group read 3 or more books a month.
  75% of our group like thunderstorms.
  100% of our group love math.

Step Three:

- Complete a new table, which includes all your facts from step one and step two. This time organize your table from the least to greatest percent.

Step Four:

- Illustrate your "Who We Are" poster. Be sure to include the table you completed in step three.

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Who Are We?

Use the chart below to record interesting facts about members of your group. For each fact compare part of the group to the whole. Express each comparison as a common fraction, decimal fraction and as a percent.

Fact	Part of group	Total number in group	Fraction	Decimal	Percent

Day 2 Journal Penny Chart Activity

How many pennies would there be in a stack that is as tall as you are? If you lie on the floor, how many pennies laid side by side would be as long as you are?

Activity Instructions:

In Visit the MegaPenny Project site for information about pennies. On day sixteen, discuss these facts about sixteen pennies.

The height of a stack of sixteen pennies is one inch.

The length of a row of sixteen pennies laid side by side is one foot.

Extensions:

1. Use eyedroppers to drop water one drop at a time onto a penny. Predict how many drops the penny will hold. Gather all the predictions and make a stem-and-leaf plot. Then conduct the experiment. Compare this to a dime, nickel, or quarter.
2. How many pennies can be stacked before they topple over? Predict how many pennies will stack. Then conduct the experiment. Gather everyone's experimental data and graph. How close were the predictions? What was the mean, median, mode and range?
3. If "a = $.01, b = $.02, c = $.03, d = $.04, and so on" can you find a word that costs exactly $1.00? (example: telephone) Which classmate has the most expensive name? the least expensive?
4. How much money would you make in a day, if you received one penny every time your heart beat? Run in place for 2 minutes and calculate the amount again.
5. Look at the date on a penny. Use reference materials to find out what was happening in history 100, 200, and 300 years before that penny was made. Then use your imagination to predict what will be happening in the future 100, 200 and 300 years after that penny was made.
6. Using a penny as the unit of measure, estimate and determine how much money would be needed to frame the door of the classroom. Which is the most expensive door in the school? the least expensive?
7. Estimate and measure the distance from one door to another using a penny as the unit of measure. How expensive is the trip? Try it again using a dollar as the unit of measure.
8. What protection does money have against counterfeiting?
9. Redesign a dollar bill. Use pictures cut from a magazine. Who would you use to replace George Washington?
10. Which United States Presidents have their picture on money?
11. If you start with a penny and each day double your money, how many days will it take before you are a millionaire?

Online Resources:

The MegaPenny Project. http://www.kokogiak.com/megapenny/

http://www.scopesys.com/anyday/
At this web address students can find events throughout history that happened on any day of the year.

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Day 2 Journal Penny Chart Activity

P	P	P	P						
P	P	P	P						
P	P	P	P						
p	p	p	p						

On the **sixteenth day** shade in sixteen squares of the hundreds chart for sixteen pennies. Record the value of the pennies in four ways.

Use 4 strips of four to cover the sixteen shaded squares. Show that 25 strips of four cover all 100 squares so 16/100 = 4/25.

When the sixteenth penny is placed on the chart, record:

Money	$.16
Decimal fraction	.16
Percent	16%
Common fraction	16/100
Lowest-term fraction	4/25

Visit the MegaPenny Project site for information about pennies.
http://www.kokogiak.com/megapenny/

These are just a few of the interesting facts that can be found there.

The height of a stack of sixteen pennies is one inch.

Arrange sixteen pennies into a square array. Measure and find the area of the square that could be drawn around them.

_____

Estimate what part of the square is not covered by pennies. _____

The Statue of Liberty is constructed of sheets of copper hung on a steel frame. The copper is 3/32 inch thick. Is this thicker than a penny?

The length of a row of sixteen pennies laid side by side is one foot.

Comparisons of Parts to the Whole

Instructor Page

Often, during the school day, students are confronted with data in their studies. For example, they may look at the high and low temperatures for a particular location over time, look at population growth in certain areas over time, or count the number of school lunches served on "pizza day". As they gather data from reference sources, they also begin to realize that data is often given in estimates, and, therefore, different sources list slightly different information. They must begin to make decisions on what data sources are reliable and the accuracy of the data. They need to begin to realize that mathematics can help them organize and make sense of that data. When students understand that they can control data, they begin to see the power of mathematics at work.

During a unit on the Civil War often students note the number of casualties during the major battles of the War. Without a way to organize this data the information is often meaningless to students.

Step One:

- Gather information about the number of troops and the number of casualties for the North and the South for the battles listed. Casualties are the total number of soldiers killed, wounded, or missing in a battle. (Use the web addresses listed or use another reference source.)
- Create a spreadsheet or calculate the percent of casualties during each battle by making a fraction and turning it into a percent. For example, at Shiloh 44,968 troops were engaged in the battle for the South. There were 10,699 casualties for the South at Shiloh.

$$\frac{\text{\# of Casualties for the South}}{\text{Total \# of troops for the South}} = \frac{10{,}699}{44{,}968}$$

10,699 divided by 44,968 equals .2379247
.2379347 rounded to the nearest percent is 24%.

24% of the Southern troops that entered the battle of Shiloh were killed, wounded, or missing in the battle.

Step Two:

- Get a 100 square and shade in 24% of the square. The shaded squares represent the number of soldiers that were killed, wounded, or missing for every 100 soldiers that entered the battle.

Step Three:

- In your journal write about how it must have felt to be a soldier entering a battle.
- Look at the chart listing the number of troops and the number of casualties. Then look at the information you calculated using percents. What are the advantages and disadvantages of presenting the information these two ways? Which do you think is more helpful? Why?

Extensions:

⇒ Use a reference source to determine the percent of the United States Presidents that were born in each month.

⇒ Tell what percent of the United States Presidents have been assassinated.

⇒ Find the percent of the states of the U.S. that have a capital name starting with the same letter as the name of the state. For example, the capital of Illinois is Springfield. This state would not work because the capital and the state do not begin with the same letter.

Literature Connection:

The Boys War by Jim Murphy
So You Want to be President by Judith St. George

NOTE: Information about the major battles of the Civil War is in the appendix to this module.

Participant Page

Battles of the United States Civil War

Use percents to compare statistics for the North and the South for these battles of the Civil War. Use a spreadsheet or a calculator to create this chart and to calculate the percents of troops killed during each battle

Use the internet or other reference sources to fill out this chart on the American Civil War Battles.

Battle	Dates	North		South		Percent of Casualties During Battle	
		Number of Troops	Number of Casualties	Number of Troops	Number of Casualties	North	South
Fort Sumter 1861							
Manassas/ Bull Run 1st							
Shiloh							
Antietam/ Sharpsburg							
Gettysburg							
Atlanta							
Battle of your choice							

Use these internet addresses or other reference sources to fill out the chart on the American Civil War Battles.

http://www.americancivilwar.com/cwstats.html
http://americancivilwar.com/statepic/alpha.html

M2T2

Instructor Page

Encourage students to describe some of the patterns that they observe with algebraic expressions.

Water Container Problem

This problem asks students to record all of their observations. There are several possible strategies.

- When the container is placed flat on the table the water inside takes the shape of the container. The top and bottom bases are squares and the lateral faces are rectangles. The volume of the water could be determined and the areas of the six faces could be found.
- When the container is tilted, the shape of the water changes. Students record the shapes they see and may want to find volume or areas as they look for rules. What quantities increase and which decrease?
- As the container is tilted more or less, which shapes or quantities change and which stay the same?

Allow at least 30 minutes for the groups to observe and record.

Encourage students to make statements that summarize a relationship. For example, "As the container is tilted more the area of the top water surface increases."

As the groups share their observations with the class, organize their rules. Record all the observations about the heights of the water on a sheet of chart paper. Use another chart for observations about the top surface area. Another chart might include observations of shapes of lateral faces. Discuss how different student groups observed similar relationships.

Extension:

Tip the flask so that only one vertex is on the table. Make new observations and statements that summarize relationships.

Resources:

The Water Flask Problem

The Open-Ended Approach: A New Proposal for Teaching Mathematics, Becker and Shimada, NCTM, Reston, VA, 1997. pps. 10-22

M2T2

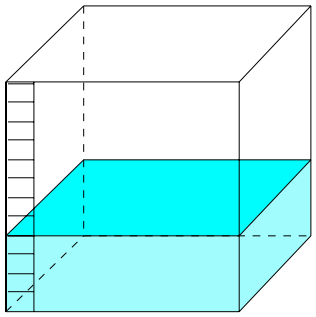
Participant Page

Water Container Problem

The edges of the rectangular prism have been marked in centimeters and the container has been partially filled with water. Place the container on a table. Then tilt it keeping one edge of its base fixed. Watch the shape and size of the water as the container is tilted more or less. Find as many rules concerning these shapes and sizes as possible. Record what you see.

What changes when the container is tilted? What stays the same?

Work with your group to describe any



Observations:

1.

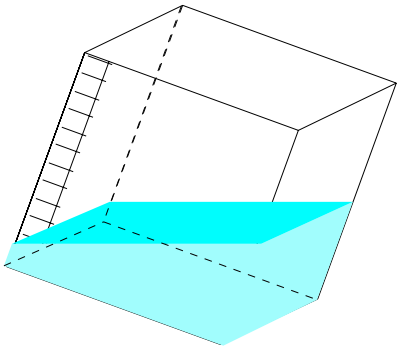
2.

3.

4.

5.

6.



7.

8.

9.

10

The Water Container Problem

The Open-Ended Approach: A New Proposal for Teaching Mathematics, Becker and Shimada, NCTM, Reston, VA, 1997

M2T2

Instructor Page

This activity applies proportions to the idea of scale. The Statue of Liberty needs scissors that are scaled to the size of her hand. Her eyeglasses must be proportional to her face.

Review the lesson on skip charts to help with using proportions.

Accessories for the Statue of Liberty

Students develop a meaning of proportions by making connections with something with which they are familiar. Everyone knows that babies have tiny feet and tiny hands, just as basketball players have large feet and large hands. Their bodies are proportional. They "fit". It would be funny to see a basketball player that wears size 22 shoes with hands the size of a fourth or fifth grader.

First step:
Bring several different sizes of shoes to class. Start with a pair of baby shoes. Let students look at the shoes then sketch the size of hand the baby would have that could fit into the tiny shoes. Do the same with children's shoes and with adult's shoes. (If you can find them, students love to try on size 22 shoes.) Students notice that the shoes and hands are proportional. As shoes get bigger, so would the hands.

Step two:
Discuss that shoes and hands are not the only things that get larger as people grow. Children have smaller eyeglasses than adults do. They also have smaller rings and scissors. They wear smaller mittens. What else? Ask the class to think about the size shoes and hands the Statute of Liberty would need. Explain that they could use proportions and mathematics to find the size of pencils, crayons, scissors, eyeglasses, or ring that the Statue of Liberty would need.

Extension: Design and conduct a survey to find the average shoe sizes for each grade level in your school. Organize your information and present in to the class.

Online Resources:

- http://www.nps.gov/stli/prod02.htm

Participant Page

Accessories for the Statue of Liberty

Have you every wondered what size pencil the Statue of Liberty would use? Or what size ring she would wear? How about what size scissors she would need, or what size eyeglasses she would wear? These problems can be solved using proportions.

STEP 1: Use reference sources to complete this chart. Use met-

Item	Statue of Liberty's Measurements	Your Measurements
Length of the nose		
Distance across the eye		
Length of the index finger		
Length of the right arm		
Distance from the heel to the top of the head		

STEP 2: Measure these items. Use metric measurements.

	Measurement	
Item	Length	Width
Your pencil		
Your scissors		
A new crayon		
Eyeglasses		

M2T2

Instructor Page

Accessories for the Statue of Liberty (Continued)

Extension:

When Stuart Little was born he was only 2 inches tall. He was about the size of a mouse. Use proportions to determine the size, then make a paper model of a pencil, crayon, scissors, or eye-glasses for Stuart Little.

Resources:

- Kids Discover "Immigration" April 1998
- World Famous Landmarks by Cynthia Adams

Literature Connection:

- Kids at Work: Lewis Hine and the Crusade Against Child Labor by Russell Freedman
- Immigrant Kids by Russell Fredman
- How Much is a Million? By David M. Schwartz
- Is A Blue Whale the Biggest Thing There is? By Robert E. Wells
- Gateway to Freedom by Jim Hargrove

M2T2

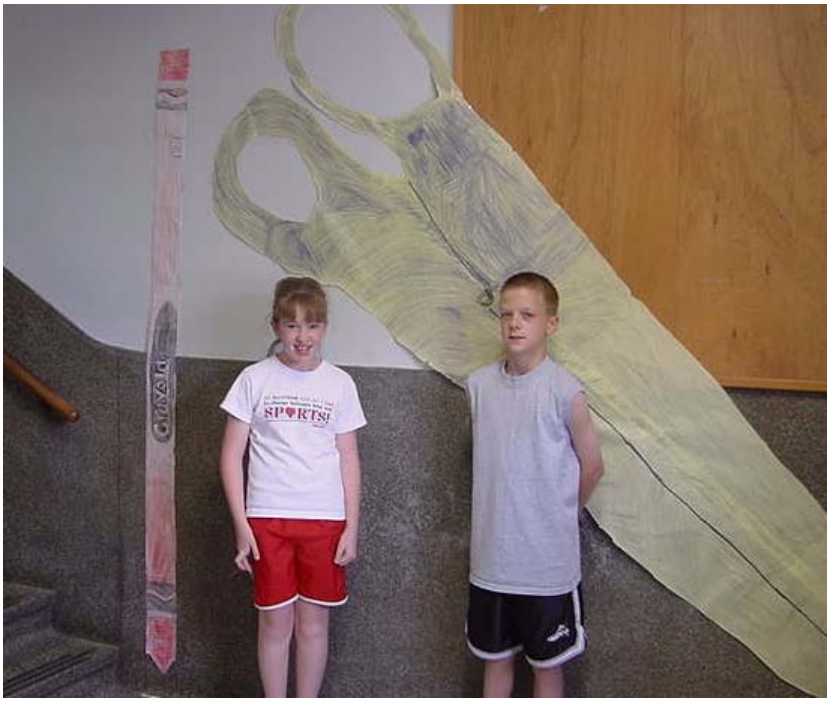
Participant Page

Accessories for the Statue of Liberty (Continued)

STEP 3: Use proportions to determine the sizes of these items for the Statue of Liberty.

	Measurement	
Item	**Length**	**Width**
Your pencil		
Statue's pencil		
Your scissors		
Statue's scissors		
Your new crayon		
Statue's new crayon		
Your eyeglasses		
Statue's eyeglasses		

STEP 4: Make a life-size paper model of a pencil, scissors, eyeglasses, or a crayon that would be the right size for the Statue of Liberty to use.



Hint:
Use proportions to enlarge the items to fit the Statue of Liberty.
For example:
If your pencil is 17 cm long and your index finger is 8 cm long, you can use the ratio of your pencil to your finger to calculate the length of a pencil that would be the right size for the Statue of Liberty's finger. The Statue's finger is 244 cm long. We can use a proportion like the one below to find the size of her pencil.

$$\frac{yourpencil}{yourfinger} = \frac{Statuespencil}{Statuesfinger}$$

$$\frac{17}{8} = \frac{x}{244}$$

It takes a skip of 30.5 to go from 8 to 244. So a same size skip from 17 would make the Statue's pencil 518.5 cm.

Small numbers

Instructor Page

Often students do not see a need for small decimals. They have no experience with small decimals, and they don't understand that when you multiply a number by a decimal the answer is a smaller number than the number you started with. They know when you multiply whole numbers you get a larger number. They need practice to see that multiplying decimals give smaller not larger numbers for answers. This activity allows students to use very small decimals and discover what happens when multiplying decimals.

Activity Instructions:

⇒ For this activity students will convert foreign currency into United States dollars using the current exchange rates. The students can "visit" any country in the world with the help of the web addresses below. If these are unavailable, the daily currency exchange rates for many countries are printed in the business section of most newspapers . Students can use maps or atlases to locate countries.

⇒ Provide items for students to "buy" with price tags on each item. The price tag has a number, but no symbol to designate the currency. Tell the students the currency of the item depends on where they go to buy the item. In Japan, for example, the price would be in yen, but in Italy it would be in lira.

⇒ For each item
- Select a country to visit.
- Find the name of that country's currency and pretend you are in that country and wish to buy an item priced with that country's currency.
- Multiply the price of the item you wish to buy by the currency exchange rate for that country. This will convert the foreign currency into United States dollars. Round to the nearest cent.
- Keep a record of how much you spend. You have only $100 U. S.

Online Resources:

http://www.expedia.com/pub/agent.dll?qscr=curc
http://moneycentral.msn.com/investor/market/rates.asp
At either of these web addresses students can compare and convert U. S. dollars into any foreign currency. Using current exchange rates, they will explore and use small decimals.
http://geography.about.com/es/worldmaps/
At this web site students can click on An Atlas of the World Maps and locate any country in the world alphabetically.
http://www.ecstaticfuturist.com/MiscInfo/numbers.html
Students can find prefixes for very small and very large numbers here.

Extension:

How small is a milligram?
- Take two lumps of clay each with a mass of one gram. Smear one of the grams onto a piece of paper and label it: One Gram
- Divide the other gram into ten sections. Smear one of these sections onto a piece of paper and label it: One Decigram
- Take one of the ten decigrams and divide it into ten sections. Smear one of these onto a piece of paper and label it: One Centi-gram.
- Take one of the centigrams and divide it into ten sections. Smear one of these onto a piece of paper and label it: One milligram.
- In your journal, write about things that are small enough to be measured with milligrams.

Literature connections:

What's Smaller Than a Pygmy Shrew? By Robert E. Wells
Alexander, Who Used to be Rich Last Sunday by Judith Viorst
Where the Sidewalk Ends "Smart" by Shel Silverstein

M2T2

Participant Page

Spend $100

You have 100 U.S. Dollars to spend.
What would you buy? You can't spend more than $100.

Country and Continent	Currency	Item to be purchased	Price in foreign currency	Exchange Rate	Decimal Answer	Rounded to U.S. Dollars
Example: Japan - Asia	Yen	Book	5.37 yen	.00781	.0410307	$.04

In your journal write about this activity.

1. What did you discover about multiplying decimals?
2. What are the differences and similarities when multiplying whole numbers and multiplying decimal numbers?

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Appendix A

Solutions and Other Hints to the Statue of Liberty Facts Activity

The Statue of Liberty stands on Bedloe's Island in Upper New York Bay. She holds her lighted torch high above her head and has been a symbol of freedom and a promise of new life since 1886 .

In 1876 Frederic Auguste Bartholdi announced that he wanted the people of France to give a gift to the United States to celebrate their centennial (100 year) anniversary of the Declaration of Independence. This gift would become the Statue of Liberty. Bartholdi had actually started building the Statue in 1875. He finished it 9 years later in July, 1884 . It took him about the same number of years to build the statue as the average fourth-grade student has been alive.

9
100
1875
1876
1884
1886

The Statue of Liberty was a joint effort between the United States and France. The American people were to build the pedestal and the French people were responsible for the Statue. In both countries the money was raised by donations.

After it was finished the Statue made the ocean journey from France to the U. S. For the trip the Statue was in 350 pieces and packed in 214 crates. It took four months to put the Statue back together on its new pedestal. On October 28, 1886, the dedication of the Statue of Liberty took place in front of thousands of spectators. She was a centennial gift ten years late.

four
ten
214
350
1886

The Statue was constructed of 300 sheets of thin (3/32 inch) beaten copper that were hung on an iron frame. Over the years the weather caused the iron to react with the copper and caused corrosion. In the 1980 's the iron armature rods were replaced with stainless steel rods. The workers removed a few of the old rods at a time and replaced them with exact copies. It took a year to replace 10,000 feet of rods. If these rods were laid end to end, they would be almost as tall as seven Sears Towers.

3/32
seven
300
1980
10,000

At the time she was built, the Statue was the largest in the world. The height from the base of the statue to the torch is 151 feet and from the ground beneath the pedestal to the tip of the torch is 306 feet. The length of her hand is $16\frac{5}{8}$ feet, and the length of her index finger is 8 feet. The distance across one of her eyes is 2.5 feet, while the length of her nose is 4.5 feet. That's about the same as the height of an average fifth grade student. The total weight of the copper in the Statue is 62,000 pounds (31 tons), and the weight of the steel is 250,000 pounds (125 tons). Her total weight, including all parts of the structure (copper, steel, glass, etc.), is 225 tons.

2.5
4.5
8
$16\frac{5}{8}$
31
151
225
306
62,000
125
250,000

Visitors climb 354 steps to reach the crown or 192 steps in order to reach the top of the pedestal. In most houses there are twelve steps between floors. That means the climb to the top of the pedestal is the same as sixteen floors.

There are 25 windows in the crown. They symbolize gemstones found on earth. The seven rays of the Statue's crown represent the seven seas and seven continents.

seven
seven
seven
twelve
sixteen
25
192
354

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Appendix B

A person stands near the Statue's foot.

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Appendix C

Use Skip Charts to Solve Proportions

Write about a situation that could be described with this skip chart.

	0	4	8							36
Cost in cents										

Write about more situations that could be described with skip charts. Make the skip charts and solve.

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Appendix D

American Civil War Battle Statistics
Commanders and Casualties

BATTLE	DATE	COMMANDER	STRENGTH	CASUALTIES
Mill Springs	1/19/1862	Crittenden Thomas	6,000 4,000	533 262
Fort Donelson	2/12/1862	Floyd Grant	21,000 27,000	15,067 2,832
Shiloh	4/6/1862	A.S. Johnston Grant	40,000 63,000	10,694 13,047
Fair Oaks Seven Pines	5/31/1862	Joesph Johnston McClellan	42,000 42,000	6,134 5,031
Seven Days' Battle Oak Grove Beaver Dam Creek Gaine's Mill Savage's Station Glendale Malvern Hill	6/25/1862 to 7/1/1862	Lee McClellan	95,000 91,000	20,614 15,849
Second Manassas	8/28/1862 8/30/1862	Lee Pope	49,000 76,000	9,197 16,054
South Mountain	9/14/1862	Lee McClellan	18,000 28,000	2,685 1,813
Antietam	9/16/1862 9/18/1862	Lee McClellan	52,000 75,000	13,724 12,410
Perryville	10/7/1862 10/8/1862	Bragg Buell	16,000 37,000	3,396 4,211
Fredericksburg	12/11/1862	Lee Burnside	72,000 114,000	5,309 12,653
Murfreesboro	12/31/1862	Bragg Rosecrans	37,000 43,000	9,865 11,577
Chancellorsville	5/1/1863	Lee Hooker	57,000 105,000	12,764 16,792
Champions Hill	5/16/1863	Pemberton Grant	20,000 29,000	3,851 2,441
Vicksburg Campaign	5/18/1863	Pemberton Grant	22,000 46,000	31,275 4,550
Tullahoma	6/24/1863	Bragg Rosecrans	34,000 65,000	1,634 560
Gettysburg	7/1/1863	Lee Meade	75,000 83,000	28,063 23,049

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American Civil War Battle Statistics
Commanders and Casualties page 2

BATTLE	DATE	COMMANDER	STRENGTH	CASUALTIES
Chickamauga	9/19/1863	Bragg Rosecrans	68,000 58,000	18,454 16,179
Chattanooga	11/23/1863	Bragg Grant	46,000 56,000	6,667 5,824
Wilderness	5/5/1864	Lee Grant	61,000 102,000	11,400 18,400
Spotsylvania	5/12/1864	Lee Hancock	52,000 100,000	12,000 18,000
Cold Harbor	6/1/1864	Lee Grant	62,000 108,000	2,500 12,000
Petersburg	6/15/1864	Beauregard Grant	42,000 64,000	2,970 8,150
Peach Tree Creek	7/20/1864	Hood Thomas	19,000 20,000	2,500 1,600
Atlanta Hoods Attack	7/22/1864	Hood Sherman	37,000 30,000	8,000 3,722
Deep Bottom	8/14/1864	Lee Hancock	20,000 28,000	1,700 2,901
Cedar Creek	10/19/1864	Early Sheridan	18,000 31,000	2,910 5,665
Franklin	11/30/1864	Hood Schofield	27,000 28,000	6,252 2,326
Nashville	12/15/1864	Hood Thomas	23,000 50,000	4,462 3,061
Appomattox Campaign	3/29/1865	Lee Grant	50,000 113,000	 10,780

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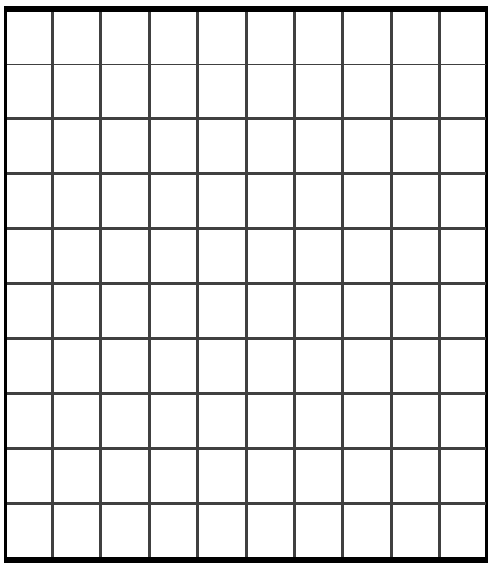
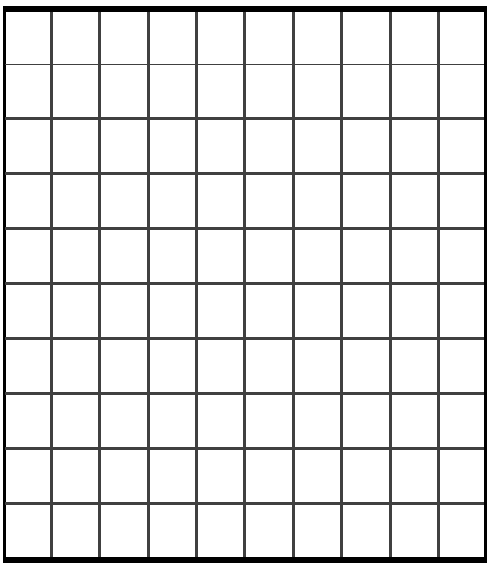
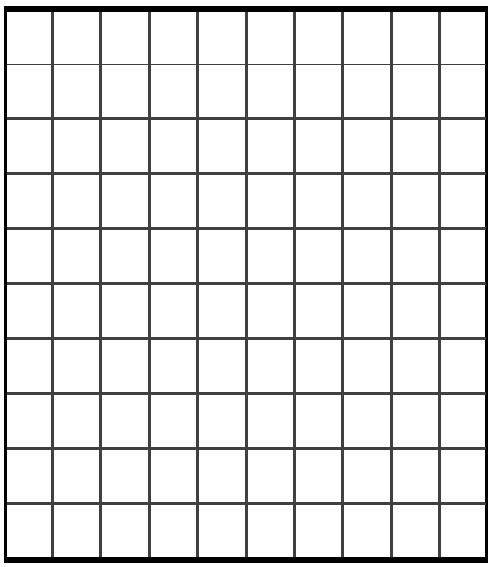
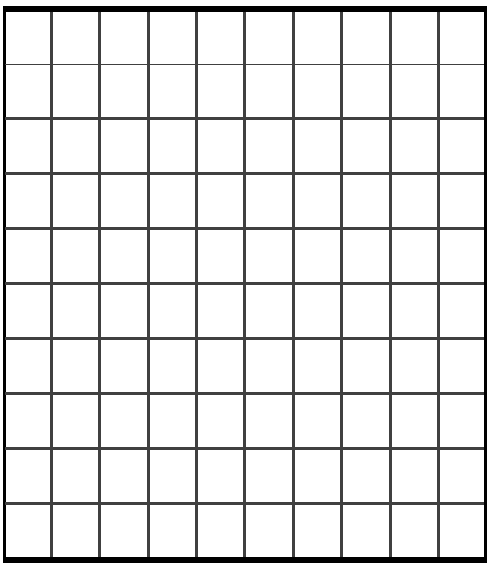
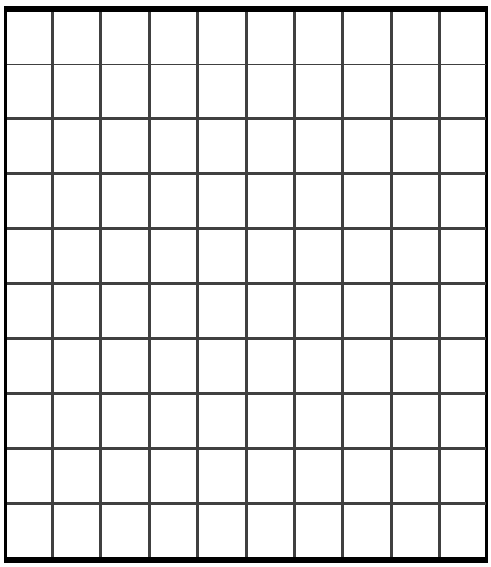
Appendix E

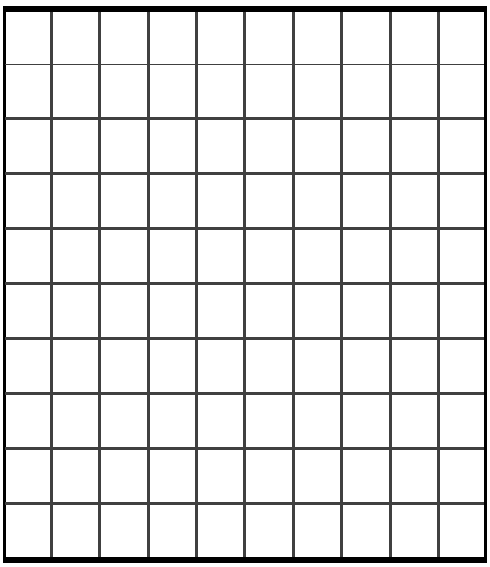












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Number Sense