# Aligning Preservice Elementary Mathematics Preparation with the 2000 Content-Area Standards for Educators

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Community College and University Colleagues:

It was apparent at the June 2001 Illinois Mathematics Teacher Educators meeting in Springfield that many of us are concerned about how we are going to adapt our mathematics curriculum for preservice elementary and middle school teachers to incorporate the *Content-Area Standards for Educators* adopted by the Illinois State Board of Education in June 2000. We'll also need to prepare our students for the state-wide assessment that will begin in 2003. A third concern is the need to better coordinate the mathematics courses for preservice elementary teachers offered at community colleges and universities. Although considerable work has been done on articulation, meeting the 2000 Content-Area Standards for Educators may require a realignment.

At Eastern Illinois University we have recently completed an examination of the mathematics guidelines for elementary teachers in the Content-Area Standards and we offer what follows as a springboard for discussion about a preservice elementary and early childhood mathematics curriculum that meets the new state standards. We've divided the required standards into three groups which form the basis of three content courses, designated here as Elementary Mathematics Content I, II, and III. All the courses are presumed to address Standards 1–5: Communication, Problem Solving, Reasoning, Connections, and Technology. Where feasible, we have incorporated some of the standards for middle school teachers with the intention that the courses could meet standards for middle school preparation. We also include a brief outline of an elementary mathematics methods course.

Our purpose in distributing this material is to facilitate progress toward a state-wide consensus about the content of mathematics courses for preservice elementary teachers that will ease the transition from community college to university and provide our students the best preparation for their final assessment. Please let us know how you plan to address the increased demands of the new state standards on your campus and give us some feedback on these course outlines.

Submitted by Joyce Bishop, Allen Davis, and Joan Henn, Eastern Illinois University.

# Standards Common to All Three Courses

Elementary Mathematics Content I, II, and III are each presumed to include Standards 1–5: Communication, Problem Solving, Reasoning, Connections and Technology.

#### Standard 1: Communication

- **1A**. Understands the dynamics of working collaboratively with others.
- **2B**. Understands learning styles and learning strategies.
- 1C. Communicates verbally and in written, visual, and symbolic forms using appropriate technology.
- 1D. Creates effective learning environments where students will be able to work collaboratively in one-to-one, small group, and large group contexts.
- **1E**. Analyzes the thinking and learning strategies of all students to extend mathematical knowledge.

### Standard 2: Problem Solving

- **2A**. Understands the many strategies for problem solving.
- **2B**. Uses problem explorations and modeling to extend mathematical knowledge of all students.
- **2C**. Generalizes results of problems and extends them to other problem situations.

### Standard 3: Reasoning

- **3A**. Understands various ways of reasoning with respect to concepts, procedures, and conjectures.
- **3B**. Applies mathematical reasoning and appropriate technologies in the development of concepts, procedures, and conjectures.
- **3C**. Generalizes reasoning skills within the study of mathematics and applies or extends them to other contexts.

# **Standard 4: Connection**

- 4A. Understands the connections within the mathematics curriculum.
- 4B. Understands mathematical connections to school curriculums and to other disciplines.
- 4C. Has knowledge of the historical development of mathematics that includes contributions of men and women from various cultures.

- 4D. Develops the connections within and among the various branches of mathematics.
- 4E. Connects mathematics to other disciplines.

### Standard 5: Technology

- **5A**. Becomes familiar with the capabilities and benefits of current and emerging technologies.
- **5B**. Understands the selection, integration and utilization of appropriate technologies throughout the mathematics curriculum.
- 5C. Selects appropriate technologies for instruction.
- 5D. Integrates current technology as appropriate for instruction.

# **Elementary Mathematics Content I**

Elementary Mathematics Content I addresses number, numerations systems, and fundamental concepts of algebra, with emphasis on conjecture and justification.

#### Standard 6: Number

- 6A1. Understands number sense, including concepts of order, magnitude, mental math, estimation, place value, and senses the reasonableness of results.
- **6B1**. Understands composition, decomposition, and recomposition of numbers including place value of any base, factors and multiples, prime and composite numbers, inverses, and the application of these concepts.
- 6C1. Uses number sense to judge reasonableness of results.
- **6D1**. Chooses appropriately from mental math, paper and pencil, manipulative, and technology to perform computations.
- **6A2**. Understands decimals, fractions, ratios, proportions, and percents as ways to represent numbers and relationships.
- 6C2. Applies proportional reasoning to solve problems.
- **6A3**. Knows the properties of prime and composite numbers.
- 6C3. Uses the properties of prime and composite numbers to determine greatest common factors and least common multiples to solve problems.
- 6C4. Performs operations in any base and converts between bases.

# Standard 8: Algebra

- 8A1. Understands the order of operations.
- **8E1**. Applies the order of operations to numerical and algebraic expressions.
- 8A2. Understands the concept of variables.
- 8E2. Identifies, completes, and extends patterns and sequences.
- 8A3. Understands symbolic, numeric, and graphical representations of mathematical situations.
- 8E3. Represents mathematical situations symbolically, numerically, and graphically.
- **8A4**. Understands the properties and operations of real number including exponents, in an algebraic context.

- 8E4. Applies properties and operations of real numbers in an algebraic context.
- 8D1. Understands the properties of the real number systems.
- 8D2. Understands the properties of sets and Venn diagrams.
- 8D3. Understands the properties of complex and modular systems and their applications.

# Standard 9: Geometry

- 9C1. Understands inductive and deductive reasoning.
- 9C2. Understands the appropriate uses of different types of proof.
- **9F2**. Constructs inductive, deductive, and indirect arguments and explains the difference among them.

# **Elementary Mathematics Content II**

Elementary Mathematics Content II focuses on measurement and geometry with emphasis on conjecture, justification and proof.

### Standard 7: Measurement

- **7A1.** Knows how to measure length, area, volume, capacity, time, temperature, angles, weight and mass.
- **7B1**. Uses customary, metric, and non-standard measures.
- 7B2. Selects and uses appropriate tools to perform above measurements.
- **7A2.** Has knowledge of non-standard and emerging units such as graphic screen pixels and font size.
- **7A4**. Understands how changing one measure of a multi-dimensional object may affect other measures.
- 7C5. Explains how changing one measure of a multi-dimensional object impacts other measurements.
- 7A5. Understands conversion factors as they apply to dimensional analysis.
- **7B3**. Estimates measurement.
- 7B4. Determines acceptable measures of accuracy and calculates relative error.
- 7C1. Measures objects using both direct and indirect measurement.
- 7C2. Uses formulas to aid in indirect measurement.
- 7C3. Applies measurements and formulas to irregular shapes, regions, and solids.
- **7C4**. Reads and interprets scale drawings.
- 7C6. Reads and interprets topographical maps and architectural drawings.

#### Standard 8: Algebra

8C2. Understands concepts of distance, area, volume, and shapes that lead to limits.

### Standard 9: Geometry

**9A**. Understands terminology, properties of two- and three-dimensional shapes, and the relationships among them.

- **9E1**. Translates between two- and three-dimensional representations of the same figure including the use of coordinate geometry and graph theory.
- 9C1. Understands inductive and deductive reasoning.
- **9F2**. Constructs inductive, deductive, and indirect arguments and explains the difference among them.
- **9F1**. Makes and identifies mathematical conjectures and provides justification to support or refute conjectures using
- 9C2. Understands the appropriate uses of different types of proof.
- 9D1. Recognizes relationships and patterns in geometric figures.
- **9D2.** Uses characteristics of geometric figures including symmetry, congruence, and similarity to recognize, identify, build, draw, describe, analyze, and categorize two- and three-dimensional figures and tessellation.
- 9D3. Applies geometric concepts to solve practical applications.
- **9E2**. Uses manipulative, Euclidean geometry, coordinate geometry, transformational geometry, and appropriate technology to model mathematical concepts and solve problems.
- 9D4. Explains the relationships between perimeter, area, and volume of similar figures.
- 9E3. Generates solids of revolution from two-dimensional figures.
  - (a) manipulatives
  - (b) constructions
  - (c) interactive technology
  - (d) paragraph and two-column proofs.

# **Elementary Mathematics Content III**

Elementary Mathematics Content III extends algebra content to include functions and algebraic links with geometry and calculus, and addresses probability and statistics, with attention to conjecture and justification.

### Standard 8: Algebra

- **8A1**. Understands the order of operations.
- 6B2. Understands operations with algebraic expressions and has knowledge of symbolic manipulators.
- 8E1. Applies the order of operations to numerical and algebraic expressions.
- 8A2. Understands the concept of variables.
  - 1. [8E2.]Identifies, completes, and extends patterns and sequences.
- 8A3. Understands symbolic, numeric, and graphical representations of mathematical situations.
- 8E3. Represents mathematical situations symbolically, numerically, and graphically.
- **8B1**. Understands the symbolic, numeric, graphical and verbal representations of relations and functions and the relationships among them.
- 8F1. Represents functions and relations in symbolic, numeric, graphical, and verbal forms.
- 8A4. Understands the properties and operations of real number in an algebraic context.
- 8E4. Applies properties and operations of real numbers in an algebraic context.
- 8A5. Understands a wide range of modeling applications involving graphs, tree charts, and other visual representations of data with multiple dimensions.
- **8B2**. Understands polynomial and exponential functions.
- 8C1. Understands concepts of rates of change and patterns that lead to limits.
- 8E3. Represents mathematical situations symbolically, numerically, and graphically.
- 8E5. Solves number sentences involving variables.
- 8E6. Translates word situations to algebraic sentences and solves them using algebra.
- 8E7. Uses equations and inequalities as a means of solving practical applications.
- **8F2**. Finds and uses slopes and intercepts to construct, analyze, and interpret graphs of equations and inequalities.

### Standard 9: Geometry

- **9B**. Knows Euclidean and non-Euclidean geometry, coordinates geometry, graph theory, and transformational geometry and the relationships among them.
- 9C2. Understands the appropriate uses of different types of proof.
- 9E6. Recognizes and uses relationships that exist between algebra and geometry.
- **9F1**. Makes and identifies mathematical conjectures and provides justification to support or refute conjectures using
  - (a) algebraic, coordinate, and transformational methods

# Standard 10: Probability/Statistics

- 10A. Understands how to collect, organize and represent data to answer questions.
- **10A1**. Understands how to create tables, graphs, charts, pictures, and other visual representations.
- 10E1. Creates tables, graphs, charts, pictures, and other visual representations of a set of data.
- **10A2**. Understands simple random sampling and recognizes bias.
- **10E2**. Collects simple random samples and recognizes sample bias.
- 10A3. Understands line to best fit.
- 10E3. Uses visual techniques for finding, interpreting, and applying the line of best fit.
- 10A4. Understands data trends and curves to best fit.
- **10B**. Understands summary statistics.
- 10B1. Understands measures of central tendency, variation, and position.
- **10F1**. Uses measures of central tendency and variation to describe a set of data.
- 10B2. Understands common distributions.
- 10F2. Uses common distributions as appropriate to solve problems.
- **10D**. Understands counting techniques and probability.
- 10D1. Understands fundamental counting principles, combinations, and permutations.

- 10H1. Uses the fundamental counting principles, combinations, and permutations.
- 10H6. Determines probabilities involving combinations and permutations.
- 10D2. Understands probability and odds of events.
- 10H2. Determines the probability and odds of events.
- 10D3. Understands dependent and independent events.
- 10H5. Computes probabilities for dependent and independent events.
- 10D4. Understands the use of random variables to solve problems.
- 10H7. Generates and interprets probability distributions for random variables.
- **10H3**. Analyzes problems situations (e.g. fairness of games, lotteries).
- **10H4**. Creates simulations to analyze simple theoretical or experimental probabilities
- 10C. Understands how to interpret and predict the results of data analysis.
- 10C1. Understands the design of observations and experiments to answer questions.
- 10C2. Understands the concept of reliability and validity.
- **10C3**. Understands correlation and regression techniques.
- 10E. Poses questions and collects, organizes and represents data to answer those questions.
- **10F**. Calculates, explains, an interpret summary statistics.
- 10G. Predicts, calculates, interprets, and applies the results of data analysis.
- 10G1. Develops a hypothesis based on a question or problem of interest and devises a plan for the collection of data.
- 10G2. Uses simple survey and sampling techniques.
- 10G3. Collects, records, organizes, displays, summarizes, and interprets data.
- 10H. Demonstrates and uses counting techniques and probability.

# **Elementary Mathematics Methods**

An elementary mathematics methods course could address the following topics:

- The pervasive role in mathematics instruction of the Process Standards emphasized by the Illinois Goals for Learning and the National Council of Teachers of Mathematics: problem solving, mathematical communication, reasoning and proof, connections, and mathematical representation, and technology.
- Understanding how children learn mathematics, planning for instruction, and appropriate use of manipulatives.
- The interaction of assessment and instruction.
- Early number concepts, number sense, numeration, and place value.
- Whole number operations: operation sense, basic facts, and algorithms.
- Rational numbers: fractions, decimals, ratios, proportion and proportional reasoning, and percent.
- Data and statistics.
- Probability.
- Patterns, relationships, and early algebra.
- Geometry and measurement.

While it's certain that all of these topics cannot be thoroughly "covered" in any single course, it is important to provide preservice teachers with a coherent view of elementary mathematics curricula and instill the notion that they can and must continue to learn about the effective teaching of mathematics.