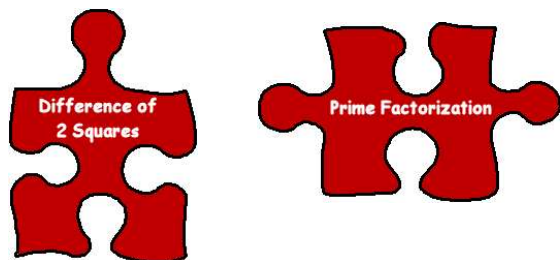


Topic: Factoring and Graphing The Difference of Two Squares:



Introduction:

The product of the sum and difference of the same two terms equals the difference between the squares of those terms. Also shown as:

The difference of Two Squares:

$$a^2 - b^2 = (a+b)(a-b)$$

This is a method for factoring a polynomial. Recall that we can also factor a number down to its prime factors using Prime Factorization and divisibility rules.

We can demonstrate this relationship with a variety of materials:

Students should attempt as many of these methods as possible to verify that the relationship:

$$a^2 - b^2 = (a+b)(a-b) \text{ is indeed true.}$$

First let's look at this numerically. At least for an initial level of verification. Let's take for example $a = 9$, and $b = 4$.

$9^2 - 4^2 = 81 - 16 = 65$ or putting it into the terms we'd like to use:

$a^2 = 81$ (with $a=9$) and $b^2 = 16$ (with $b=4$) and finally $(9 + 4)(9 - 4) = 13 * 5 = 65!$

WOW! It really works! Try it out with several other numerical examples.

Activity: Try to factor these as illustrated in the example above:

1. $49 - 4 =$ _____

2. $25 - 64 =$ _____

3. $11^2 - 12^2 =$ _____

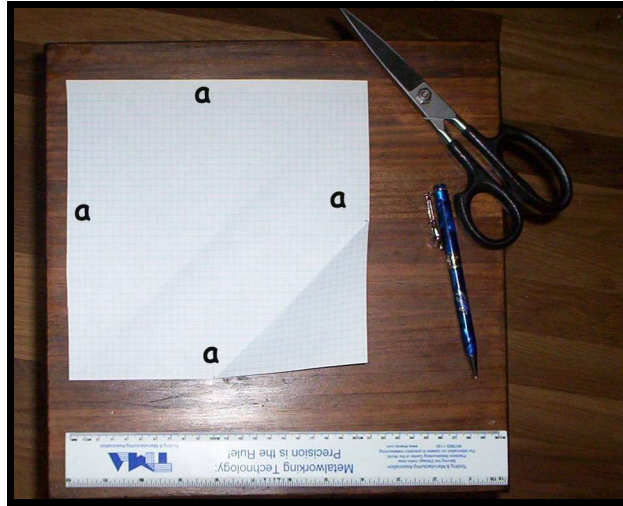
4. $225 - 16 =$ _____

Here is a collection of photos of how the difference of two squares can be modeled with

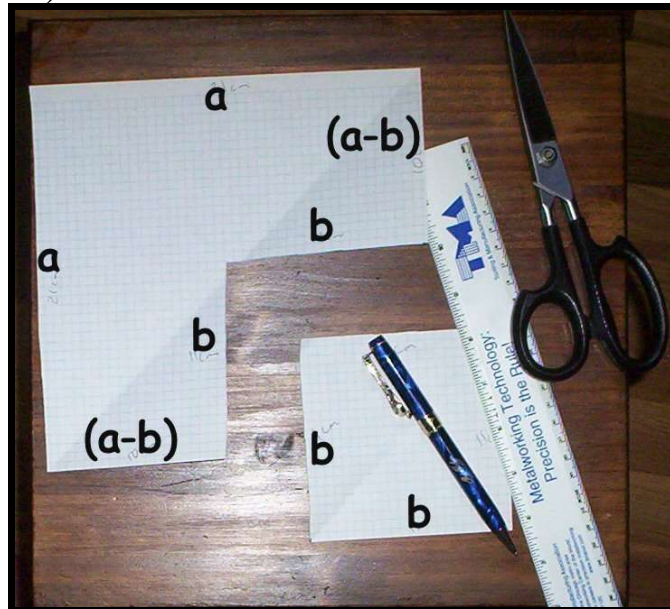
paper & scissors:

A movie showcasing this model can be found online at www.mste.uiuc.edu/dildine/times/diff2sq.gif

Step One: Obtain or create a square sheet of paper. Size does not matter but a larger square will be easier to use for our model. Each side of this piece measured 21 cm. Which gives us an area of 441 cm^2 . Label these sides (a) and label the area as a^2 .

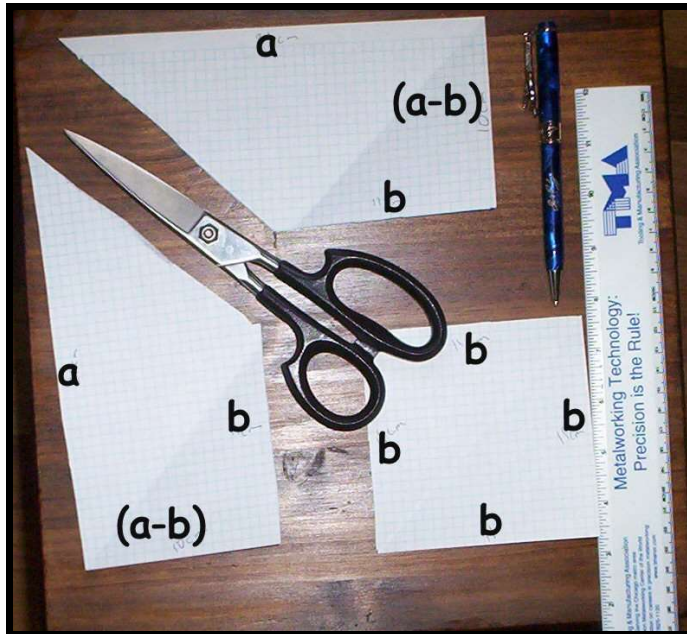


Step Two: With a trusty ruler and “Super Shears” from CutCo (any scissors will do) We measure and remove a smaller square labeled “ b^2 ” (in this case $b = 11 \text{ cm}$. Sides which gives us 121 cm^2 area). As shown here:

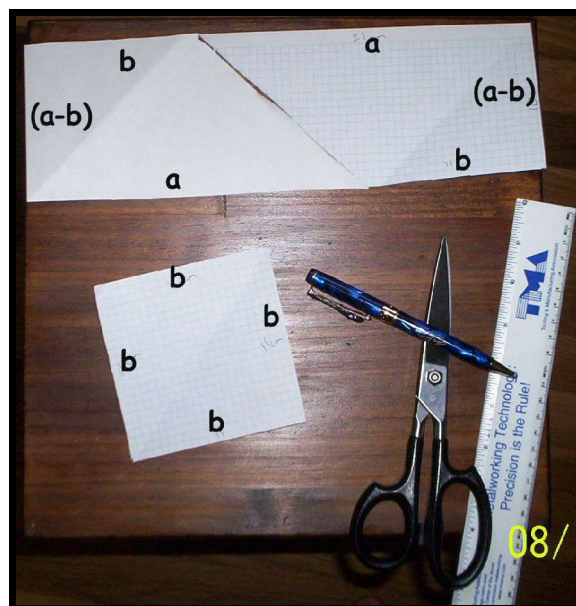


The new smaller square becomes b^2 . The sides of the original square are labeled a and (a-b) since we removed the length of b from each side. .

Step Three: We cut the diagonal of the remaining portion of square a.



Step Four: We rearrange the remaining parts of the square “a” to form a new figure (rectangle) that we can measure the area of.



Looking at this new figure (rectangle) and using the measurements we had before we can begin to track back through our cutting to determine that physically, $a^2 - b^2 = (a+b)(a-b)$. Where $(a+b)$ is the length of the new rectangle and $(a-b)$ is the width. Using the formula for the area of a rectangle we discover that removing square b we end up with $(a+b)(a-b)$ as the area of the new figure. We remember that the area of a rectangle is $L \times W$ or in this case length $(a+b)$ and width $(a-b) = ???$

Looking at our example we have $a^2 = 441$ and $b^2 = 121$ so $a^2 - b^2 = 320$ so we should have

$(a - b)(a+b) = 320$. Plugging in our values we have $(21 - 11)(21+11) = (10)(32)=320$. It works!

An online demonstration of this exists at www.mste.uiuc.edu/dildine/diff2sq.htm

For this figure the results are summarized as follows in cm and cm^2 :

a (original square)	Area of a	b (removed square)	Area of b	(a-b)	(a+b)	(a-b)(a+b)
21	441	11	121	10	32	320

Use the spreadsheet located at <http://www.mste.uiuc.edu/dildine/times/diff2q.xls> to check your values, and experiment with additional values. Fill in values in the color filled columns to see values for $a^2 - b^2$.

In addition to the online movie, there are a number of other technological resources that you can visit. Some using Java Technology and some using Geometer's Sketchpad Technology.

View these sites to get spreadsheets and GSP files as well as Java Applets.

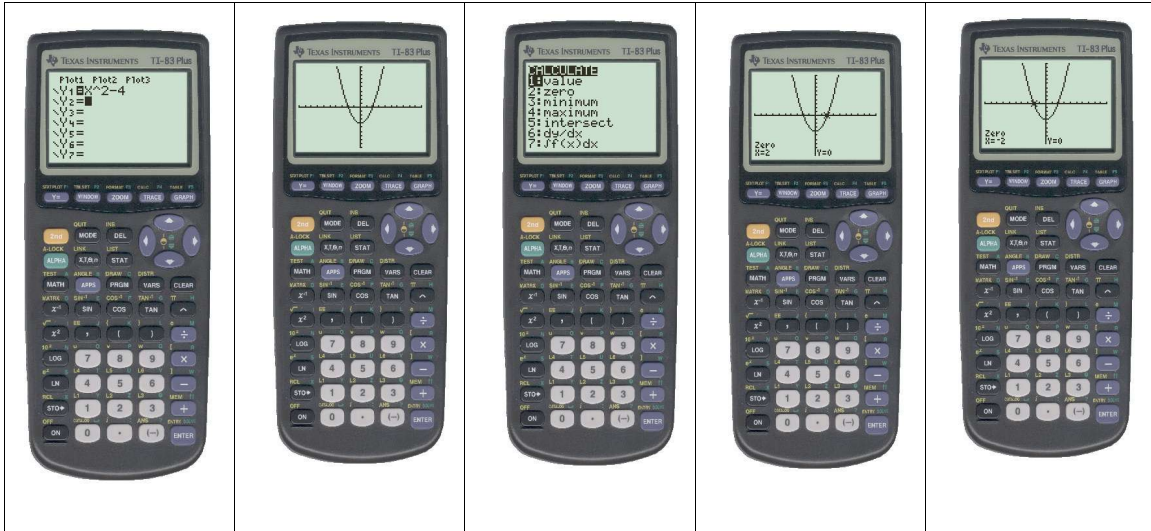
<http://www.mste.uiuc.edu/dildine/times/diff2sq.html>

Activity: Factor the following using the difference of two squares (if not explain why):

1. $x^2 - 36$ _____
2. $16y^2 - 49z^2$ _____
3. $9h^2 - 100$ _____
4. $q^2 + 25$ _____
5. $31^2 - 6^2$ _____
6. $121 - x^2$ _____

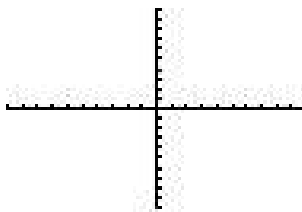
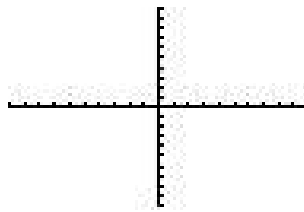
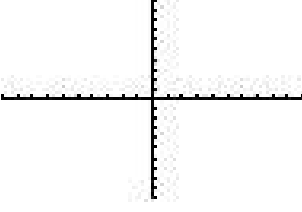
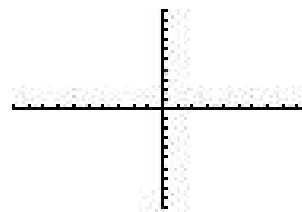
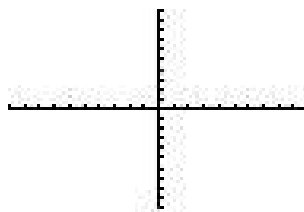
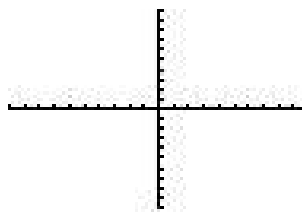
The end result of our finding the difference of two squares is finding the solution to an equation. We can use a graphing calculator as shown below to find the solution to the equation $X^2 - 4 = 0$. Note that this is an evident difference of 2 squares.

Graphing the equation $X^2 - 4$ we can use the TI-83+ to find the zero values. We know from the difference of two squares that $X^2 - 4$ can be factored into the following $(X - 2)(X + 2) = 0$. We can eyeball this to guess that our X values would be 2 or -2 to make it equal to zero or we can graph the equation and find where the zero values are (where the graph crosses the X-axis). This is shown in the following set of screenshots.



First we enter our equation in the Y= Screen then we graph it. Then we can hit 2nd Calc to calculate our zero values by choosing our Left and Right bounds and giving a guess within those bounds. You can see for our equation $Y=X^2 - 4$ or $(x-2)(x+2)$ we have zero values of -2 and 2. This can be done with any of the following equations, as well.

Activity: Draw the graphs in the boxes below identifying their zero values (if any):

<p>1. $y = 4x^2 - 16$</p> 	<p>2. $y = 81 - x^2$</p> 	<p>3. $y = x^2 - 9$</p> 
<p>4. $y = 4 + x^2$</p> 	<p>5. $y = x^2 - 49$</p> 	<p>6. $y = 49 - x^2$</p> 

Activity: Do the graphs look different when you rearrange the terms (Ex. $X^2 - 9$ vs. $9 - X^2$)?

Are they still the difference of two squares?

Activity: What would it look like if you graphed equations like this together? What is the relationship between their “zero values”?