M2T2

How far, how fast? How many, how much?

Number Sense

STATE GOAL 6:

Demonstrate knowledge and use of numbers and their representations in a broad range of theoretical and practical settings.

Statement of Purpose

The ability to reason quantitatively helps us to function effectively in many aspects of our everyday lives. The intuitive feeling for handling quantitative information is what we often refer to as "number sense." There are four aspects of number sense which form the foundation of this module and promote the ability to reason quantitatively. These four aspects are:

- ***Quantities***—being able to deal comfortably with different types of quantities and extremely large or small numbers that are unfamiliar;
- ***Representations***—understanding that different quantities often require different labels and are best represented by different types of graphs, tables, or expressions;
- ***Relationships*** between quantities—being able to make comparisons, to form ratios and to use proportional reasoning;
- ***Patterns***—being able to recognize patterns, and how these patterns can be represented, can help us to discover trends in numerical data, and thus help us reason inductively.

Number sense is an important component of many content areas and processes of mathematics, including algebra, geometry, probability/statistics, and measurement. Increased number sense also enhances our ability to reason mathematically, to solve problems, and to communicate mathematical understanding..

Highlights

In this unit, we will explore the four aspects of number sense listed above. The unifying activity of the module is to build a *theme-based poster* that demonstrates those understandings through written communication, graphs, and illustrations of numerical facts about the chosen theme.

Connections to the Illinois Learning Standards.

Standard 6.A.—Demonstrate knowledge and use of numbers and their representations in a broad range of theoretical and practical settings. This standard is addressed throughout the module in the poster activity, the Sears Tower (or Gateway Arch) activity, and all the additional activities. The variety of uses of numbers and their representations is a theme of this module.

Standard 6.B.—Investigate, represent and solve problems using number facts, operations (addition, subtraction, multiplication, division) and their properties, algorithms and relationships. Activities such as "How Big is the Base of the Pyramid of Giza" address this standard.

Standard 6.C.—Compute and estimate using mental mathematics, paper-and-pencil method, calculators and computers. Estimation is key to the Sears Tower activity. Computers and/or calculators are recommended for graphical displays in the colored candies activities.

Standard 6.D.—Solve problems using comparison of quantities, ratios, proportions and percents. The poster activity and the pie chart use this. As students draw pictures of the base of the Great Pyramid, they will be doing comparisons.

Table of Contents

Note: Appendices are printed only on the odd pages. This is done to make photocopying easier. That is, each participant should have a copy of all the odd numbered pages. While the instructors should have a copy of all the pages.

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Materials

- Large Poster Board
- A variety of fine point colored markers
- Various colors of card stock paper
- Grid paper (6 squares per inch)
- Rulers, compasses, protractors, yard sticks
- Masking tape
- Pie plates
- Individual bags of **colored candies**
- Scissors
- Graphing Calculators
- Internet access
- Geometer's Sketchpad® or Cabri® software
- Microsoft Excel or ClarisWorks or other spreadsheet software
- Various reference books such as a world almanac, encyclopedias, *The Mathematics Teachers' Book of Lists,* etc.

Section of the Module	*Objectives for each Part of the Module*
Day 1	• *To develop an understanding of how numbers are used to describe and compare objects, including as numerical benchmarks to obtain a sense of magnitude* • *To develop an intuitive understanding of scale and equivalent ratios* • *To develop and refine strategies for estimating and checking reasonableness of results* • *To recognize, understand, and use various representations for large numbers (e.g., exponential, scientific)*
Day 2	• *To understand different ways of organizing the same data* • *To understand how different types of graphs can be used to represent numerical information* • *To create graphical and visual representations that are appropriate for a set of data*
Day 3	• *To understand how to make comparisons by subtraction* • *To understand how to make ratio comparisons* • *To use ratios in conversion of units* • *To use ratios as proportions in scaling*
Day 4	• *To recognize various types of numerical patterns* • *To generate various numerical patterns, or sequences* • *To create interesting visualizations of numerical patterns* • *To recognize trend patterns in data* • *To pose appropriate questions about numerical information and trend data*

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Instructor

Goal of Day 1:

A primary goal of number sense is to build an understanding of how numbers are used in many different aspects of life. The activities on Day 1 ask students to recognize that numbers are used for different purposes and require different labels depending on their use.

Opening Activity: Sears Tower or Gateway Arch; How high can you go, how far can you see?

NOTE: Solutions to this activity are in the appendix to this module.

Context: This activity utilizes familiar places and people to help students make sense of numbers. Several numerical facts are stated, except that the numbers are removed from the statements. Students must estimate reasonable heights, weights, and distances, to choose which number belongs in each blank, choosing from the list of numbers on the right. Some analysis of the statements is required in order to be able to choose the correct number (e.g., _____ feet per minute is equivalent to _____ mph). Students must also understand scientific notation (e.g., 16,100 is equivalent to 1.61×10^4).

Activity Instructions

⇒ Distribute copies the Sears Tower activity to all students. Have students work independently for 10 – 15 minutes filling in the blanks with the numbers. Remind students that the numbers are grouped for each paragraph.

⇒ Group students in groups of three to compare and discuss their answers. Ask them to verify their answers by showing their calculations, where appropriate (e.g., Michael Johnson's speed in feet per minute and miles per hour).

⇒ When most groups have finished, ask some groups to report their results and procedures for verifying their answers.

Discussion of Math Content and Related Questions:

- Discuss the different ways that we use numbers in everyday life. Can you separate the different numbers in this article into different categories? Which ones? Why did you decide to categorize the numbers in this particular way?
- For example, sometimes numbers are used as ***counts***—there are 110 floors, you can see 4 states. Other times, we use numbers as ***measures***—it weighs 225,000 tons and is 1454 feet high. Also, we use ***ordinal numbers***, such as ***dates,*** to identify where something appears in an order. For example, ***6/3/1998 is the 3rd day of the 6th month of the year 1998.***
- Often, we use two different numbers as benchmarks to make comparisons that help us grasp unusually large or "fantastic" quantities—it would take 224 Michael Jordans standing on top of each other to equal the height of the Sears Tower. The use of these kinds of comparisons to give "reality" to extreme quantities leads to the idea of *ratios* (which we will explore in Part 3).
- Another important aspect of this activity and discussion are the use of labels: What labels are appropriate for a particular use of a number as a count? As a measure? As a ratio? Why do we write some numbers as scientific notation and other shortcuts such as "1.5 million"?

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Participant Page

The Sears Tower: How High Can You Go, How Far Can You See?

In the following paragraphs about the Sears Tower, fill in the blanks with the appropriate quantities that complete the information. The numbers to be used in the paragraph are located in the list to the right:

The Sears Tower, opened in _____ , took _____ years and $_____ to build. The building has _____ gross square feet of floor space, and a combined weight of _____ tons or _____ lbs. This weight is equivalent to the mass of about _____ average size men, almost all the men who live in Chicago. The average wind speed in the Windy City is _____ mph and the average sway of this massive building is approximately _____ inches from true center.

The Sears Tower is _____ feet high - that´s about _____ Michael Jordans standing on top of each other. On a clear day, you can see _____ states - Illinois, Indiana, Wisconsin and Michigan. Visibility from the Skydeck is approximately _____ to _____ miles. Including the twin antenna towers the Sears Tower is _____ feet high. _____ Radio and Television stations broadcast from the Tower antennas.

The Sears Tower has approximately _____ bronze-tinted windows and _____ roof-mounted robotic window washing machines to clean all _____ windows. Sears Tower elevators are among the fastest in the world, operating as fast as _____ feet per minute which is over _____ mph! That is probably about the speed of the fastest bicycle rider in your class. Compare that to Michael Johnson´s fastest running speed of about _____ feet per minute (_____ mph). That´s a fast elevator!

Approximately _____ tourists visit the Skydeck each year. That is the same number of Kindergarten through 8th grade students in all of Illinois. If there are _____ students per class and _____ classes in your school, that would be _____ classes, or _____ schools, visiting the Skydeck each year.

Go to http://www.sears-tower.com for lots of information about the Sears Tower in Chicago, Illinois.

Choose from the following numbers for each paragraph:

3
6
16
1973
222,500
2,472,222
4.5×10^6
150 million
4.45×10^8

4
22
40
50
224
1454
1707

6
18
22.35
1.6×10^3
1967
16,100
1.61×10^4

25
30
2×10^3
60,000
1.5 million

M2T2

Instructor

Opening Activity: Sears Tower or Gateway Arch; How high can you go, how far can you see?

NOTE: Solutions to this activity are in the appendix to this module.

Activity Instructions

⇒ Distribute copies the Gateway Arch activity to all students. Have students work independently for 10 – 15 minutes filling in the blanks with the numbers. Remind students that the numbers are grouped for each paragraph.

⇒ Group students in groups of three to compare and discuss their answers. Ask them to verify their answers by showing their calculations, where appropriate (e.g., Michael Johnson's speed in feet per minute and miles per hour).

⇒ When most groups have finished, ask some groups to report their results and procedures for verifying their answers.

Optional Activity: Web Scavenger Hunt)

⇒ Group students ***(preferably in pairs)*** and prepare for working on computers with Internet Web access.

⇒ Distribute copies of the Sears Tower or Gateway Arch activity to each pair.

⇒ Instruct the students to use the related Web Sites to find the numerical facts represented in each of the statements, and then fill in the blanks with the appropriate numbers listed in the column on the right.

⇒ Ask students to verify their answers by showing equivalency statements, where appropriate.

⇒ When most groups have finished, ask some groups to report their results and procedures for verifying their answers.

Extension

- Another extension of the activity would be to have students ***categorize*** how the numbers in the Sears Tower activity are used. This gives students a chance to take ownership of the ideas as they sort them out in their mind. Examples of categories they might generate are: ***years and dates, numbers of an item, rates such as "miles per hour", etc.*** Discuss with them how their categories relate to counts, measures, ordinal numbers, ratios, and equivalencies. There will certainly be overlaps.
- A longer extension is to have students select a theme and create their own version of the Sears Tower/ Gateway Arch activity with their theme. This could be an additional component of their poster presentation.

Internet Resources:

- http://www.sears-tower.com is a well-designed website with the information in the article activity and much, much more for the curious. Follow the link to the Skydeck: http://www.the-skydeck.com
- U.S. Census Bureau Population Statistics - http://www.census.gov/
- Illinois School Statistics - http://www.isbe.net/research/htmls/statistics.htm
- Jefferson National Expansion Memorial- http://www.nps.gov/jeff/. The general information is at http://www.nps.gov/jeff/generalinfo.htm
- Michael Jordan - http://www.nba.com/history/players/jordan_summary.html

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Participant Page

The Gateway Arch: How High Can You Go, How Far Can You See?

By Martha Eggers

Fill in the following paragraphs about the Gateway Arch with the appropriate quantities that complete the information. The numbers to be used in the paragraph are located in the list to the right:

The Gateway Arch, designed by Eero Saarinen in _____, was completed in _____ and took _____ months to complete. This catenary curve is _____ meters wide and _____ feet tall, _____ feet more than Mark McGuire's longest home run. This height is equivalent to _____ six-foot men standing on top of each other. On a clear day the visibility radius from the observation area is _____ miles.

20
30
85
105
192
630
1948
1965

_____ cu.ft. of earth and rock were excavated to prepare the site for the foundation. The Arch itself weighs _____ tons, used _____ tons of stainless steel, and cost _____ dollars. Each year more than _____ visitors take the tram ride to the top. The tram has been operating over _____ years, traveled over _____ miles total, and carried over _____ passengers.

30
900
17,246
2.5×10^5
3×10^5
10^6
13 million
25 million

The observation platform has a maximum capacity of _____ visitors. The _____ seat passenger train's speed is _____ feet per minute, but the service elevator travels at a rate of _____ feet per minute. The passenger's round trip takes _____ minutes, and there are _____ steps in each leg from the base to the top for emergency purposes. The Arch was built in _____ sections and, to ensure that the constructed legs would meet, the margin of error for failure was _____ inch. In a _____ mile per hour wind, the Arch will deflect _____ inches at the top, but the usual sway is only _____ inch.

1/64
1/2
10
18
40
100-140
142
150
340
400
1076

The Gateway Arch is the tallest national monument in the United States and the _____ most visited tourist attraction in the world. The Eiffel Tower, however, is _____ feet taller than the Arch, and the Sears Tower is _____ feet taller. Egypt's Great Pyramid is _____ feet shorter than the Arch; and both the Washington Monument, at _____ feet tall, and the Statute of Liberty, _____ feet tall, are shorter than the Arch.

4th
180
305
354
555
824

Poster Activity for Day 1: Picking a Topic

Instructor Page

Poster-Building Activity:
The main project of the module is to build a theme-based poster and to use the poster to highlight quantitative information about that theme. The poster project could take up to two weeks to complete.

Students are asked to choose themes, or topics that are of interest to them, and are asked to search for numerical facts about that theme. The Sears Tower opening activity models such a theme and many of its related numerical facts.

Poster-Building Activity Instructions:

⇒ Give the students the sheet with instructions for choosing a theme and finding ***six numerical facts*** about that theme. Introduce the poster project and explain that this particular activity is just the beginning stage of building the poster. Over the course of the entire project, there will be several activities that allow the students to contribute information to the poster.

⇒ Participants may need guidance in choosing an appropriate theme. However, you should avoid choosing a theme for the students as much as possible. The power of the activity is in the students' ownership of the topic.

⇒ What could be a theme? Possible topics range from ***Star Wars Movies*** to ***Lions, Tigers***—and, yes!—even ***Bears***. Sports figures and teams work well, e.g., ***Michael Jordan, National Hockey League,*** or the ***St. Louis Rams***. Famous architecture works nicely as our opening activity demonstrates. One major point of this module is that almost any topic of interest that the student chooses will contain a wealth of numerical information!

⇒ Students should be given ample time to research information about the theme. The Internet, encyclopedias, almanacs, the ***Math Teacher's Book of Lists***, and CD-ROM's (such as ***Encarta, Microsoft Bookshelf,*** etc.) are all valuable resources for researching a topic. Plan time in the library for this portion of the activity.

⇒ Encourage participants to record their numerical facts on a "scrap" sheet before preparing these statements for the poster. They can create the nice versions on card stock and adhere them to the poster board or they can write them directly on the poster board.

Discussion of Math Content and Related Questions:

- While the students are choosing and researching their theme, discuss with individual students what types of numerical information are possible for a given topic.
- As students identify numerical facts about their themes, try to build connections back to the Sears Tower activity. Is the information about a count (quantity) or is it some type of a measure? What units are appropriate for this type of information? What labels are used with these units? Is there specific notation that is needed to express this information? What vocabulary and types of sentences convey this information best?
- Throughout the poster project, students should be thinking about, using, and enhancing their number sense. The mathematics underlying this number sense may have been "covered" in a previous year or earlier in the current year. However, the poster activities give students a chance to use their number sense as a way of communicating their ideas to others.

Materials:
Minimal
Poster Board
Access to encyclopedic & almanac resources
Colored card stock paper
Color markers (lots)

Optimal
Internet access

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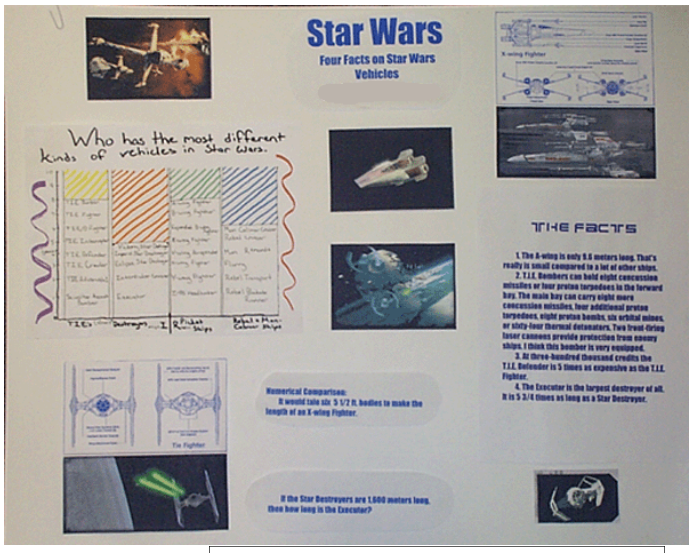
Participant Page

HOW FAR, HOW FAST? HOW MANY, HOW MUCH? THEME POSTER—IT'S YOUR PICK!

What grabs your interest???



Poster in photo by Mark Bernsen

The goal of this project is to pick a theme or topic that interests you, and to explore it from a numerical perspective.

What is an appropriate topic? Almost anything that interests you:

- ***SPORTS? It could be Michael Jordan, your favorite college sports team, Olympic sport, a professional team, or …***
- ***MUSIC? What about a style of rock band? Or the great jazz trumpet players? Or Billboards Top 100? Or…***
- ***NATURE? What about tigers or whales or even bugs? Or…***
- ***MOVIES? The Star Wars series, or Indiana Jones adventures, or Independence Day? Or…***
- ***POLITICS? The 2000 Presidential Elections , or…***
- ***ALMOST ANY TOPIC that interests YOU!!!***
  Don't limit yourself!

Got a topic in mind?? Good—because that's the first step!
Here are the general instructions for your theme-based poster.
At each stage of the poster, your instructor will give you more detailed information.

1. ***Choose a topic that interests YOU, and research it carefully.***
   Try to find as much numerical information about the topic as you can.
2. ***Write six sentences that contain numerical facts related to your theme.***
   Try to include as much variety as possible in the types of numerical information you use in your sentences.

Note: **There are 7 steps to completing the Theme Poster.**
You have just completed Steps 1 – 2.
Steps 3 – 7 will be completed in later parts of the module.

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Instructor Page

An important aspect of number sense is knowing how to represent quantitative information graphically or visually. One goal of these activities is to explore the different ways that we can organize and present data.

Materials:

Minimal

- Bags of colored candies
- Paper plates
- Graph paper
- Colored Markers
- Rulers
- Compasses
- Protractors

Optimal

- Spreadsheet software
- Internet access

Opening Activity for Day 2: All about colored candies

Context: *This activity is a short open-ended exploration. It is useful both (1) to assess students' abilities to organize data and (2) to review various types of graphs/charts and their appropriate uses.*

Activity Instructions

⇒ Distribute bags of colored candies. Ask students to estimate how many are in the bag. It is important to record on a chart, overhead, or the board ***each student's estimate.*** Then, head on to the next step. You will return to these estimates later.

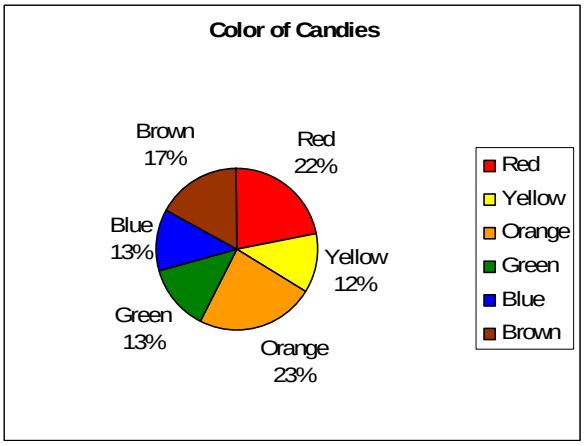
⇒ Give each student a paper plate. Ask everyone to open the bag and dump the candies onto the plate and then to ***"organize"*** the candies on the plate. Give the students about five minutes to do this.

⇒ By what criteria can the data be "organized"? Discuss the different ways students chose to organize the candy on their plate and ways they could represent it on paper with a graph.

⇒ The students then complete the chart on the next page and create a graph that represents the data.

⇒ Remember that the number of candies per package is not constant. Also, the distribution of colors will probably vary from bag to bag.

Pie charts are used to compare parts of a whole against the whole. The pie chart makes it easier to compare information about one color of candy to the whole bag.



Internet Resources:

These web sites offer information about some popular colored candies. You can find information about color distributions, nutrition, product size and other facts.

- http://us.mms.com/us/about/products/
- http://www.hersheys.com/products/
- http://www.skittles.com/products/index.jsp

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Participant Page

What About Colored Candies ?

Take a bag of colored candies and dump them onto a paper plate. Take a couple of minutes and organize them however you wish. In the space below, describe how you have organized the candies and what information you learned from organizing them in this way:

Describe how you organized the colored candies:

Complete the information in the table below.

COLOR of candies	NUMBER IN BAG	FRACTION OF TOTAL	PERCENTAGE OF TOTAL
TOTAL			

Use the information in the table to create a graph or chart that represents the data. Draw your graph below. Use appropriate tools. Be neat.

Write 3 questions that can be answered by the graph you created.

_____

_____

_____

List any other types of graphs that you could use to represent the candy data:

_____

_____

_____

_____

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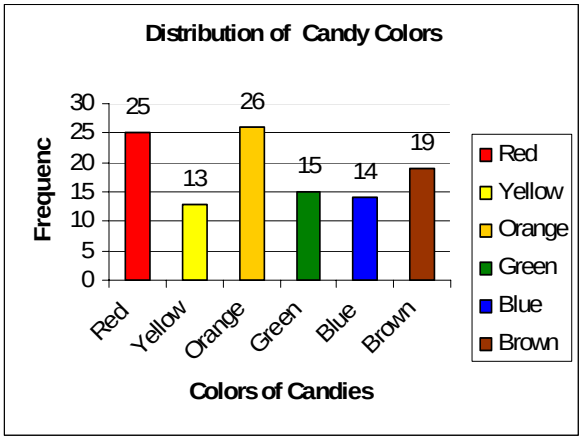
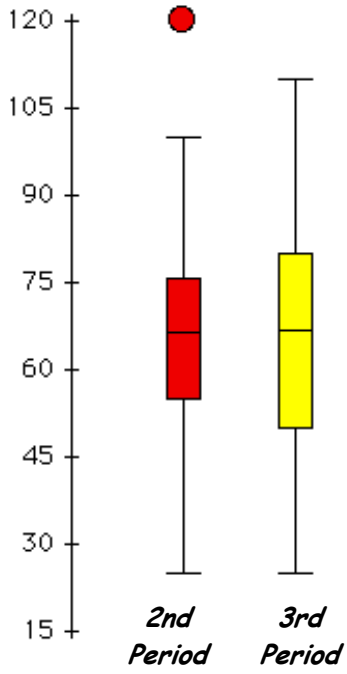
Opening Activity: Day 2 (continued)

Instructor

Bar Graphs are used to compare different parts of a whole; in the case of colored candies, it emphasizes comparisons between the different colors.



Number of Candies in a Bag



Box and Whiskers Plots are graphs that demonstrates the ***RANGE*** of values in a set of data; in the beginning, we asked the class to estimate the number of candies in a bag. The plots on the left show the range of estimates for two classes.

- ***In 2nd period, the guesses ranged from 25 to 100 with one stray student guessing 120. This extreme value is called an OUTLIER. The median guess was somewhere between 60 and 70 and the middle 50% of the guesses were between 55 and 75.***
- ***In 3rd period, the guesses ranged from 25 all the way up to 110. There were no extreme values, or outliers. The median value was also between 65 and 70 and the middle 50% of the guesses were between 50 and 80.***

Stem and Leaf Plots are used to look at all of the individual values in a set of data but in a very compact way. This type of graph is useful to show central tendency, especially ***MODE—the most frequent response.***

In the 2nd period class, most of the estimates of the number of candies in a single bag were between 60 and 80. The most frequent guess, the mode, was 75.

Stems—Each number is a multiple of ten.	*Leaves*—Each number represents a data point.
1	
2	5 5
3	4
4	1 5 5
5	0 5 5 5
6	0 0 0 5 5 8 8
7	5 5 5 5 6 6 8
8	
9	0 0
10	0 0 0
11	
12	0
13	

Internet Resource:

- ***http://www.mste.uiuc.edu/m2t2/number/m-ms.xls*** is a Microsoft Excel spreadsheet that provides the students with ways to organize and graph data.

Participant Page

Extended Activities With Colored Candies

Use a spreadsheet or chalkboard chart to input all of your class's estimates for the number of candies in their bag, as well as the corresponding actual totals for each student's bag of candies.
Record the data you collected on the number of each color of candy.

Next, go to one of the candy websites, and find all the information that you can about the different candies, and how they are made.

- http://us.mms.com/us/about/products/
- http://www.hersheys.com/products/
- http://www.skittles.com/products/index.jsp

Use the candy information from your classmates and the websites. Your goal is to create each of the following types of graphs using appropriate information.

- ❖ Bar Graph
- ❖ Pie Graph
- ❖ Stem and Leaf Plot
- ❖ Box and Whiskers Plot
- ❖ Scatter Plot of Two Variables

With each, provide an explanation of why that particular graph is the best choice to represent those data.

Challenge Question: What is the probability (likelihood) that the first candy you pour out of a bag of colored candies is a ***blue*** candy? Devise a plan for solving this problem and explain your strategy below.

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Poster Activity for Day 2: Making a Graph or Chart

Instructor Page

Basic graphs can be made quite creatively and even look professional using a spreadsheet or other graphing software. Encourage participants to explore with these tools of technology to create interesting graphs.

However, remember to check to see that the graph is not misleading and is appropriate for the type of data being represented.

Poster-Building Activity Instructions:

⇒ Return to the facts in ***The Sears Tower/Gateway Arch—How High Can You Go, How Far Can You See?*** activity. Discuss how some of the numerical facts could be made more meaningful if included in a graph. For example, the height of the Sears Tower could be included in a graph that also shows the heights of other tall buildings. What kind of graph would show this information best?

⇒ Ask the participants to make an appropriate graph that would include information about their chosen theme. Remind them to include ***a title, labels and accurate scales.***

Extensions & Variations:

- Have the students research their theme to find data appropriate for each type of graph discussed.
- The US Census Bureau website listed below provides detailed data—even about the very community in which you and your students live! Have the participants research for data about your community and use those data to create one of each type of graph or chart discussed in class.
- Have the participants create 3 questions that could be answered from each graph. One question should be straightforward, one should be more of an interpretation of the graph or chart, and one question should be a higher-order thinking question that requires analysis of the data.

Materials:

Minimal
- Posters (already begun)
- Access to encyclopedic & almanac resources
- Colored and white card stock paper
- Color markers

Optimal
- Spreadsheet software
- Internet access (WWW)

Internet Resource:

- U.S. Census Bureau Population Statistics – http://www.census.gov/

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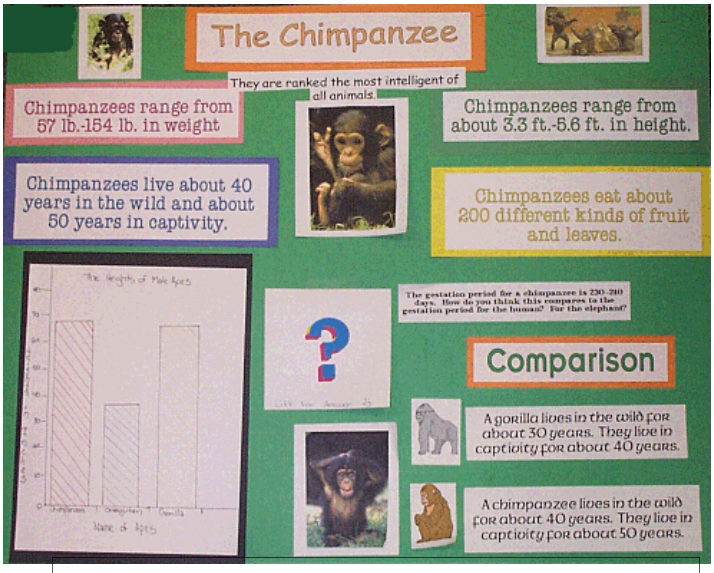
Participant Page

HOW FAR, HOW FAST? HOW MANY, HOW MUCH? THEME POSTER—IT'S YOUR PICK!



Poster in photo by Bridget Kimmey

Let's continue the poster assignment. You've chosen a topic and found interesting numerical information about that theme. Now, create an appropriate graph or chart that makes information about your theme more visual.

For example, the height of the Sears Tower could be included in a graph that also shows the heights of other tall buildings. What kind of graph would show this information best?

1. *Choose a topic that interests YOU, and research it carefully.*
   Try to find as much numerical information about the topic as you can.
2. *Write six sentences that contain numerical facts related to your theme.*
   Try to include as much variety as possible in the types of numerical information you use in your sentences.
3. ***Make a graph or chart. It can show relationships among your facts or it can show how one of your facts is related to facts of a similar theme.***

Remember to include a title, labels, and accurate scales on the graph!

Note: There are 7 steps to completing the Theme Poster.
You completed Steps 1 – 2 in the first part of the module,
and have just completed Step 3.
Steps 4 – 7 will be completed in later parts of the module.

Opening Activity for Day 3: How Big is the Base of The Great Pyramid of Giza?

Instructor Page

Goal of Day 3:

The goal of this part is to understand the different types of comparisons we use between numbers to make very large or very small quantities more familiar. Both additive and ratio comparisons will be used in the poster, and the opening activity explores the use of proportions in scaling. Additional activities are provided that focus on the use of ratios to convert units.

Context: *This activity is a short open-ended exploration that invites students to get a handle on the incredibly huge size of the Great Pyramid of Giza by creating a scale drawing.*

Opening Activity Instructions:

⇒ Hand out the worksheet on participant page. Have a clear idea of where you can use floor space in your classroom to block off a 10 ft. by 10 ft. square and have a few students do this.

⇒ Tell the participants that ***"to help us think about this problem, a scale drawing of this comparison will help."*** Use either grid paper or Geometer's Sketchpad to create a scale drawing of the 10 ft. by 10 ft. square and the base of the Great Pyramid. *(Instructions for using Geometer's Sketchpad to create the scale drawing are in the appendix.)*

⇒ If you choose to use grid paper, the participants will have to tape several sheets together. This is a great hands-on way to give them a perspective of just how immense the base of the pyramid really is.

Possible questions to ask after they have blocked off the 10 ft. by 10 ft. square and created a scale drawing: ***Could we make a square that is the actual size of the base of the pyramid? Could it be taped off anywhere inside our school? Would we need to go outside? How long would it take us to walk all around the edges of this actual model of the pyramid's base?***

Discussion of Math Content and Related Questions:

Optional: One way that you can help students get a visual grasp of the size of this pyramid is to visit the football field if it is convenient. It would take a little more than 2 and 1/2 football fields in length to equal the length of one side of the Great Pyramid of Giza.

This is an excellent opportunity to introduce or review ***some*** of the ways we use ratios:
We use ratios:

- as benchmarks to make sense of extreme values;

Example: it takes about 2½ football fields to equal one side of the Pyramid at Giza, or, in the language of ratios:

$$\frac{\text{length of pyramid base}}{\text{length of football field}} \approx \frac{2\frac{1}{2}}{1}$$

- to convert units; Example: 4.3 ft = 4.3 ft * 12 in/1 ft = 51.6
- to create scale models or drawings:
  Example: Suppose a model of the Great Pyramid uses a scale of 1/8 in = 1 foot. What will be the length of the base of the pyramid in the scale model corresponding to the 775 ft length in the real pyramid? *Solution: Set up the proportion: s/775 = (1/8)/ 12. Now cross multiply and solve for s. (Answer: ≈ 8 ft.)*

Note and discuss with the participants: the 10' by 10' marble tiles won't fit exactly into the 775' base of the pyramid. At the end you may have to use "half-tiles" (10' by 5'). What does all this tell you about the *area* of the base?

Materials:

Minimal
- 10' x10' floor space in classroom
- Masking Tape
- Ruler or Yard Stick
- Grid Paper

Optimal
- Geometer's Sketchpad

Internet Resource:

- Sketchpad file is at ***http://www.mste.uiuc.edu/m2t2/number/gizabase.gsp***

M2T2

Participant Page

How Big Is the Base of the Great Pyramid of Giza?

The Great Pyramid of Giza in Egypt has a square base that is 755 feet on each side. To help us think about how big this really is we can compare it to a square that we can measure in the classroom. Use masking tape to enclose a square on the floor of the classroom that is 10 feet on each side.

Problem: Suppose the builders of the pyramid wanted to cover the floor of the base with marble tiles that are 10' x 10'. How many times will a 10' x 10' square fit inside the base of the Great Pyramid?

Use Geometer's Sketchpad or grid paper to draw a square that is 10 units on each side. This small square will represent the square we enclosed in the classroom. Use this same scale, 10 units = 1 foot, to draw a scale model of the base of the Great Pyramid of Giza. How long is one side of your large square drawing? Compare it to the smaller drawing. Remember that the small square compares to the masking tape square in the classroom in the same way that the large square compares to the base of the pyramid. Could we make a square that is the actual size of the base of the pyramid? Could it be taped off anywhere inside our school? Would we need to go outside? How long would it take us to walk all around the edges of this actual model of the pyramid's base?
Write a paragraph to answer these questions and record the math work that you do.

Suppose that you want to create a scale model of the Great Pyramid of Giza that is five feet by five feet on the base of the Pyramid. What size of tiles would you have to use for a scale version of the 10' x 10' marble tiles to cover the base of the Pyramid model? Explain carefully.

M2T2

Poster Activity for Day 3: Making Comparisons and Using Ratios

Instructor Page

Ratios can be written in a variety of ways:

"224 Michael Jordans equals the height of the Sears Tower" can be written as

224:1

and as a fraction by either

$$\frac{1}{224} \text{ or } \frac{224}{1}$$

- ***What's missing from all of these forms above? THE UNITS! In a ratio, units are very important.***

Poster-Building Activity Instructions:

⇒ Return again to the facts in *The Sears Tower/Gateway Arch—How High Can You Go, How Far Can You See?* To introduce the third aspect of the Theme Poster project, point out the comparisons within the text that make the numerical facts related to the Sears Tower easier to understand.

⇒ *If we compare the height of Michael Jordan to the height of the Sears Tower, there would be 224 Michael Jordans standing on top of each other to equal the height of the Sears Tower. Talk about how imagining 224 Michael Jordans standing on top of each other has more meaning than thinking about 1454 feet. This is a comparison that uses a ratio.* Ask the participants to make a comparison using a ratio that makes one of their numerical theme facts more understandable.

⇒ *If we were to compare the height of the Sears Tower to the height of the John Hancock Building, it would be more sensible to compare by subtracting. The John Hancock Building is 1127 feet tall and is the third tallest building in Chicago. The Sears Tower is 327 feet taller than the Sears Tower.* Ask the participants to make an additive comparison (or comparison using subtraction) that makes another of their numerical theme facts more understandable.

⇒ Ask the participants to illustrate the comparisons that they have described on their poster.

Materials:

Minimal
- Posters (already begun)
- Access to encyclopedic & almanac resources
- Colored and white card stock paper
- Color markers

Optimal
- Spreadsheet software
- Internet access (WWW)

Discussion of Math Content:

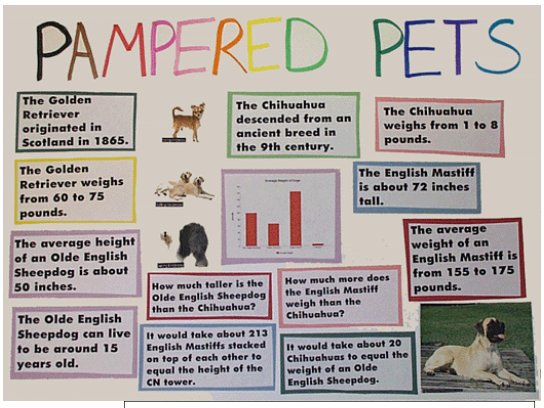
- Sometimes a ratio compares two numbers that have the same units or units that measure the same thing. The height of Michael Jordan is compared to the height of the Sears Tower. Both are measured in feet or both are measured in meters.
- Other times a ratio can compare two entirely different quantities. The fuel efficiency of a car is measured in ***miles per gallon (mpg),*** which is the comparison of the distance traveled (miles) and the fuel used (gallons).
- ***When two ratios are equal to each other, they make a proportion.*** Ratios are used to compare and proportions are used to find unknown quantities that are related to known quantities. We use ratios and the proportions formed from them, when we interpret maps, make scale drawings, or compare similar figures.
- Percentages are ratios that compare to one hundred.
- Pi, the trigonometric ratios and the golden ratio are special ratios that you and students may want to explore further.

M2T2

Participant Page

How Far, How Fast? How Many, How Much? Theme Poster—It's Your Pick!

Your theme poster is beginning to take shape.



Poster in photo by Cassie Ross

- Use an additive comparison to make one of your sentence facts more meaningful. Use phrases like "larger than" and "less than" to describe the comparison. Include a visual representation of that comparison.

- Use a ratio to illustrate or describe a comparison for a different one of your facts. Use words like "times as many" or "stacked on top of" or "laid end-to-end" to describe the comparison. Include a visual representation of that ratio.

1. Choose a topic that interests YOU, and research it carefully.
   Try to find as much numerical information about the topic as you can.
2. Write six sentences that contain numerical facts related to your theme.
   Try to include as much variety as possible in the types of numerical information you use in your sentences.
3. Make a graph or chart. It can show relationships among your facts or it can show how one of your facts is related to facts of a similar theme. Be sure to include a title and appropriate labels for your graph.

4. Use an additive comparison to make one of your numerical facts more meaningful. Use phrases like "larger than" and "less than" to describe the comparison. Include a visual representation of that comparison.

5. Use a ratio to illustrate or describe a comparison for a different numerical fact. Use words like "times as many", "stacked on top of", or "laid end-to-end" to describe the comparison. Include a visual representation of that ratio.

Note: There are 7 steps to completing the Theme Poster.
Earlier, you completed Steps 1 – 3 in the first two parts of the module, and have just completed Steps 4 – 5.
Steps 6 – 7 will be completed in the last part of the module.

Opening Activity for Day 4: Fibonacci's Crazy Quilt

Instructor Page

Goal for Day 4:

Being able to recognize numerical patterns helps us to recognize trends in numerical data. The goal of the final section is to complete the poster by recognizing trends or patterns and creating analytical questions about those trends.

Context: *This activity is a fun activity to increase familiarity with numerical patterns such as the Fibonacci sequence. The end product (called a Fibonacci X 4 quilt) is a great wall-hanger. Can be adapted to many other sequences and patterns.*

Opening Activity Instructions

⇒ This activity uses the sequence 1,1,2,3,5,8,13,21, 34,… to create a quilt square. Give participants a sheet of grid paper (6 squares per inch), one or two pieces of blank paper (that you can see through if you lay the grid paper underneath) and a ruler.

⇒ On the bottom horizontal edge of the grid paper, start in the center and mark off the sequence 1, 1, 2, 3, 5, 8 to the right of the center. Then, start at the center again, and mark off the same sequence 1, 1, 2, 3, 5, 8 to the left. Since the 1, 1 is in the center, it is a part of both the sequence design on the left and on the right. See the diagram on the participant page.

⇒ Using a ruler, draw vertical lines to divide the grid into "bar" sections. Try doing this directly on the plain white paper with the grid paper underneath.

⇒ Repeat the same process along the vertical edge of the grid paper to create horizontal lines. Draw a 3 grid unit border around the whole square. The students should now have a "patterned grid" like the model on the participant page.

⇒ Now, the participants are ready to decorate the quilt squares. Let each student decide on a color pattern for the quilt and color it. Below is a finished product hanging in a school's display case.

Materials:

Minimal
- Grid Paper (6 squares per inch)
- Rulers
- Finepoint colored pens

Optimal
- Internet access

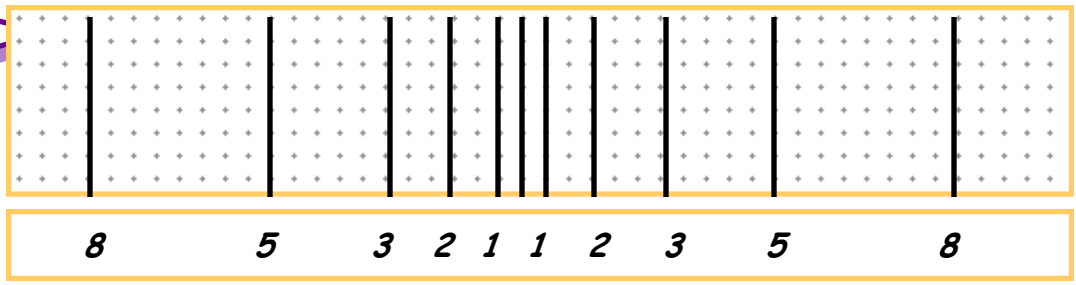
Internet Resource:

- A richer exploration of the Fibonacci sequence, put together by two middle school teachers (Kris Hightshoe and Sara Burrus) is available at ***http://www.cmi.k12.il.us/~hightshoeburrussa/index.html***

M2T2

Participant Page

Fibonacci's crazy quilt

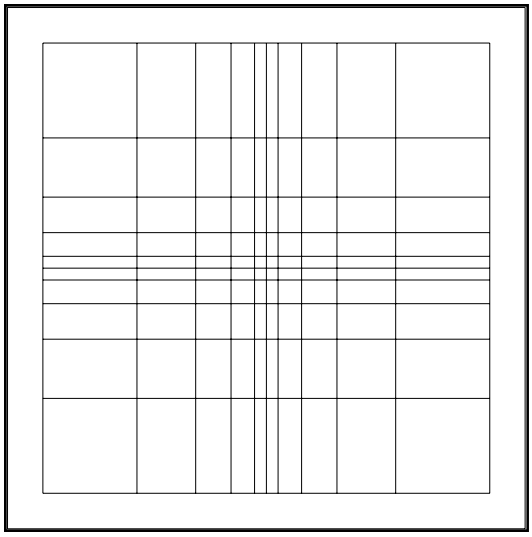


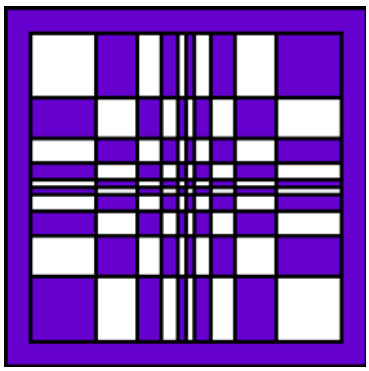
This activity explores the Fibonacci sequence in 2 dimensions. Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13, 21, 34, …
What is the rule to get the next term in the Fibonacci sequence?

What are the next 5 terms in the sequence?

Directions for creating your quilt square:

- On a sheet of grid paper, along the bottom edge, start one unit left of center and mark off—to the right—lengths of 1, 1, 2, 3, 5, 8—the first 6 terms in the Fibonacci sequence.
- Start one unit right of center and mark off the same lengths to the left. Use the same 1, 1 in the center for both the right-hand and the left-hand sequences.
- Draw vertical lines to divide the grid paper into bars.
- Do the same procedure along the vertical edge of the grid paper, and draw horizontal lines. This creates a "patterned grid". Create a 3-unit border all the way around. Using tracing paper, create a clean copy of the quilt square on white paper. It should look similar to the pattern on the right.





- Decide on a color pattern for your quilt square and color it.
- Piece your quilt square together with the other students' quilt squares to create a crazy quilt.

The idea for this activity comes directly from *Mathematical Quilts—No Sewing Required! by Diana Venters and Elaine Krajenke Ellison.*

Opening Activity for Day 4 (part 2): Patterns of Disappearance

Instructor Page

Activity Instructions: Candies and patterns of disappearance

⇒ Give each participant a bag of candies with labels or an initial on one side. Students empty the candy out of the bag onto the paper plate and count the number of candies on the plate.

⇒ In the table provided on the Participant Page, "0" in the first row of the column labeled "TRIAL" represents the zeroth trial and the number of candies in the entire bag in the first row of the column labeled "Number of Candies " represents the initial number.

⇒ Now, the data collection begins. Each participant puts all of the candies on his/her plate into a small cup, shakes the candies up, and drops them back on the plate.

⇒ Count the number of candies that have an initial or label showing and eat the ones that DO NOT have a label showing on top. This is the first TRIAL. Record the number of candies that remain on the plate in the "Number of Caandies " column. (This should be all the ones with a label showing on top!)

⇒ Repeat this procedure and continue trials until all of the candies are gone.

⇒ Next the participants graph the data collected on a coordinate axis with the x-axis representing the trial number and the y-axis representing the Number ofCandies remaining at the end of each trial.

⇒ Ask participants, "Are there any patterns that you see in the data? Does the table or the graph make the pattern easier to see?"

Materials:
Minimal
Bags of colored candies with an initial or label on one side
Paper plates
Small cups
Participant page or graph paper

Optimal
Graphing Calculators
Internet access

Discussion of Math Content and Related Questions:

- The candy is a substitute for flipping a large number of coins at once. Since the candies are ***two-sided***, shaking them and dropping them on the plate is analagous to ***flipping a coin***. There is a 50:50 chance of getting either a "head" or a "tail"—or a ***1 in 2 chance of any one piece of candy landing with the label showing!***

- This is a good activity to introduce the concept of an exponential function. They can plot the data on a graphing calculator. After discussing the concept of an exponential function, use the calculator's capabilities to find the curve of best fit.

The data of this activity reflect an exponential curve of best fit—

$y = (1/2)^x$ or $y = (2)^{-x}$

Internet Resource:

- An exploration of Exponential Fit by Ed Malczewski is available at ***http://www.mste.uiuc.edu/malcz/ExpFit/INTRO.html***

M2T2

Participant Page

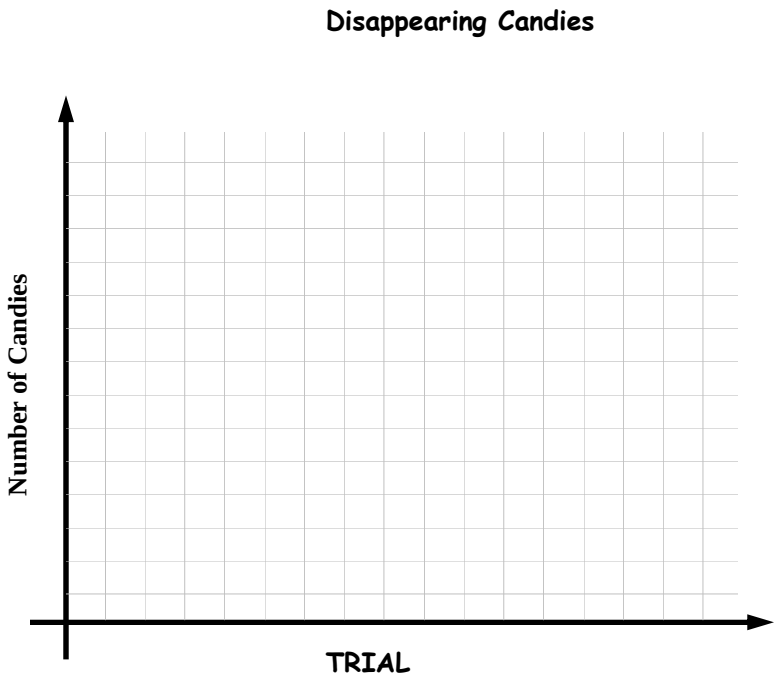
The Case of the Disappearing Candies

1. *Take a bag of colored candies and empty the candy out of the bag onto a paper plate. Count the number of candies .*
2. *In the table provided below, 0 is recorded in the first row of the column labeled "TRIAL" to represent the zeroth trial or the beginning number of candies. Record the number of candies in the entire bag .*
3. *Now, the data collection begins. Put all of the candies on your plate into your small cup, shake the candies up, and drop them back on the plate.*
4. *Count the number of candies that have an initial or label showing and eat the candies that DO NOT have a label showing on top. In the table record the number that remain on the plate in the "Number of Candies" column for the first trial. (This should be all the ones with a label showing on top!)*
5. *Repeat this procedure and continue trials until all of the candies are gone.*

What patterns do you see in the data? Does the table or the graph make it easier to see a pattern?

Draw your graph in the box below.
Use appropriate tools. Be neat.

TRIAL	NUMBER OF CANDIES
0	
1	
2	
3	
4	
5	
6	



Describe what patterns, if any, you see in your data collection. Describe how you can recognize the patterns in both the table form and the graph form of the data.

Instructor Page

Poster Activity for Day 4: Asking Questions and Preparing the Presentation

Poster-Building Activity Instructions:

- Analyzing information by looking for patterns and trends allows us to make generalizations or inferences. Graphs and comparisons help with this analysis. Have the participants revisit their poster and think about possible patterns or trends in their information.
- Ask the participants to write three questions that could be answered by analyzing the information presented by the poster.
- Give the students time to finish assembling their completed posters. Encourage their creativity.

Rubric-Building (for the Teacher's Workshop):

- Spend about 10 minutes sharing both the finished products of the posters as well as the workshop participants' final versions of the rubrics that they would use to grade this project. Look for mathematical understanding as well as completeness and appearance in their categories.

More information on assessment and a sample rubric can be found in the appendix.

Materials:

Minimal
- Posters (already begun)
- Access to encyclopedic & almanac resources
- Colored and white card stock paper
- Color markers (lots)

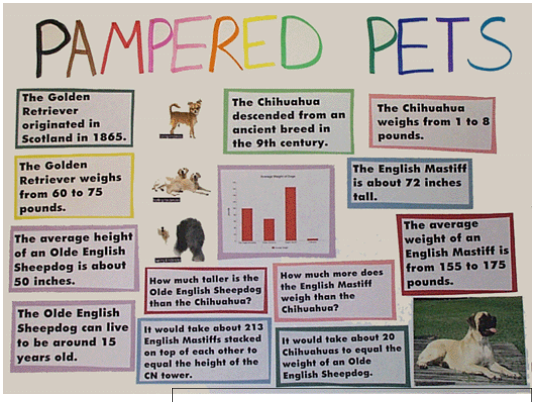
Optimal
- Spreadsheet software
- Internet access

M2T2

Participant Page

How Far, How Fast? How Many, How Much? Theme Poster—It's Your Pick!

It's time to put the finishing touches on your theme poster. Examine all of the numerical information you have presented about your theme and look for patterns and trends. You may even have to do some more research!



Poster in photo by Cassie Ross

Now, write three questions that could be answered after analyzing the information that you have provided. Ask at least one question that requires looking for a pattern or trend suggested by the graph. Put these questions on your poster. Include the answers, but hide them.

1. Choose a topic that interests YOU, and research it carefully.
   Try to find as much numerical information about the topic as you can.
2. Write six sentences that contain numerical facts related to your theme.
   Try to include as much variety as possible in the types of numerical information you use in your sentences.
3. Make a graph or chart. It can show relationships among your facts or it can show how one of your facts is related to facts of a similar theme. Be sure to include a title and appropriate labels for your graph.
4. Use a **comparison by subtraction** to make one of your sentence facts more meaningful. Use phrases like "larger than" and "less than" to describe the comparison. Include a visual representation of that comparison.
5. Use a **ratio** to illustrate or describe a comparison for a different numerical fact. Use words like "times as many", "stacked on top of", or "laid end-to-end" to describe the comparison. Include a visual representation of that ratio.
6. ***Write three questions that could be answered after analyzing the information that you have provided. Ask at least one question that requires looking for a pattern or trend suggested by the graph. Include the answers, but hide them.***
7. ***Compile all of the above into a poster presentation that is visually interesting and appealing. Neatness and creativity count!***

Note: **There are 7 steps to completing the Theme Poster.**
Earlier, you completed Steps 1 – 5 in the first three parts of the module, and you have just completed Steps 6 and 7. Congratulations!

Appendix A

Additional Activities

The following activities promote the use of ratios as benchmarks, the use of ratios to convert units, and the use of ratios to scale proportionally. Each activity could be used as an opening activity in a class session or could be extended into a full class session.

Problem: If you could stack one million pennies on top of each other in one tall stack, how high would it reach? To the top of your school building? To the top of the Sears Tower? All the way to the moon?

Instructional Tips:

- Materials needed: a few (no more than 6) pennies per team, rulers, possibly Internet access
- Allow all participants to vote for one of the three choices.
- Give pairs of students 5 or 6 pennies and a ruler and let them explore this problem.
- Hint: None of the three is a good approximation!

Have them use the Internet to find a more reasonable comparison for the height of a million pennies stacked!

Problem: If you could lay one million dollar bills end-to-end, how far would the string of dollar bills stretch? Across the city in which you live? Across the state of Illinois? Across the United States?

Instructional Tips:

- Materials needed: one dollar bill per team, rulers, possibly Internet access
- Again, allow all of the participants to vote for one of the choices.
- Give pairs of participants a dollar bill and a ruler, and let them explore the problem.
- Again, none of the above three choices is a good approximation.
- Ask the participants to devise a better estimate for the distance of a million dollar bills stretched end-to-end. Internet access can provide students with a wealth of resources. Try online map resources.

Problem: Imagine a box with a volume of one cubic yard that is completely filled with one-inch cubes. If we were to take out all of the one-inch cubes from the box, and stack them on top of each other, how high would the stack of cubes be?

Instructional Tips:

- Have students work in teams to calculate the actual height in inches and convert the height from inches to larger units where the numbers are more familiar.
- Have participants make a ratio comparison of the stacked cubes to the height of the Sears Tower.
- ***Quick thinker:*** After the participants have finished solving the stacking problem, ask them if all of the cubes stacked up could fit in your classroom? *See how many students forget that the cubes all fit in the one-yard cubic box!*

Appendix A (continued)

Additional Activities

Problem: How many tennis balls could fit compactly into your classroom. Imagine that all of the desks and movable furniture is gone, but all of the permanent fixtures remain (alcoves, closets, etc.).

Instructional Tips:

- Materials needed: 1 shoebox and 2 tennis balls per team
- Have every student contribute an estimate and record each person's estimate on the board in class.
- Then, give the students in teams of 2 (or groups of 3) a shoebox and only 2 tennis balls. No rulers or other traditional measurement tools allowed!!
- Give the students 15 minutes to devise an estimate using only the 2 tennis balls and a shoebox.
- Have each group present their conclusion and explain their methods.
- Each group, for homework, should write up their results. The written report should explain the group's method of solving this problem, their conclusion, and a justification.

Problem: Estimate the amount of time it would take you to count to one million aloud by ones. Explain carefully.

Instructional Tips:

- Materials needed: None
- Let students work in teams of two or three at most.
- As you monitor their progress, be sure that students consider in some way the variable lengths of times needed to say different numbers. For example, it takes longer to say "five hundred, sixty-four thousand, three hundred ninety eight" than it does to say "ten".
- However, it is best not to remind them of this consideration unless they never think of it. It is better that the students think of all the possible parameters. Usually, a well-placed "be prepared to explain ***everything*** carefully" will prompt them to evaluate their parameters.

Problem: What happens to the area and perimeter of a rectangle when we increase the length of each side of the rectangle by a scale factor of 2 (i.e., double all the sides)? How about when we triple the length, that is, by a scale factor of 3? Can you develop a rule for how the area and perimeter of a rectangle changes for any scale factor?

Instructional Tips:

- Materials Needed: Geoboards, rubber bands
- Place the students in teams of 2 or 3.
- Give each student on a team a different size and shape of rectangle to build on the geoboards using rubber bands.
- Have the students record data in a table. They can pool their information and use the table to make a conjecture about a general scale factor.

Extension: Repeat the problems for boxes, i.e., what happens to the volume of a box when you double or triple each of its dimensions? What implications does this have for the expected heating and air-conditioning costs of a house or apartment?

Appendix B

Solutions and other hints to the Sears Tower Activity: How High Can You Go, How Far Can You See?

The Sears Tower, opened in _1973_ , took __3__ years and $_150 million_ to build. The building has _4.5 million_ gross square feet of floor space, and a combined weight of _222,500_ tons or _4.45×10^8_ lbs. This weight is equivalent to the mass of about _2,472,222_ average size men, almost all the men who live in Chicago. The average wind speed in the Windy City is _16_ mph and the average sway of this massive building is approximately _6_ inches from true center.

The Sears Tower is _1,454_ feet high - that´s about _224_ Michael Jordans standing on top of each other. On a clear day, you can see _4_ states - Illinois, Indiana, Wisconsin and Michigan. Visibility from the Skydeck is approximately _40_ to _50_ miles. Including the twin antenna towers the Sears Tower is _1,707_ feet high. _22_ Radio and Television stations broadcast from the Tower antennas.

The Sears Tower has approximately _16,100_ bronze-tinted windows and _6_ roof-mounted robotic window washing machines to clean all _16,100_ windows. Sears Tower elevators are among the fastest in the world, operating as fast as _1,600_ feet per minute which is over _18_ mph! That is probably about the speed of the fastest bicycle rider in your class. Compare that to Michael Johnson´s fastest running speed of about _1,967_ feet per minute (_22.35_ mph). That´s a fast elevator!

Approximately _1.5 million_ tourists visit the Skydeck each year. That is the same number of Kindergarten through 8^{th} grade students in all of Illinois. If there are _25_ students per class and _30_ classes in your school, that would be _60,000_ classes, or _2000_ schools, visiting the Skydeck each year.

Choose from the following numbers for each paragraph:

3
6
16
1973
222,500
2,472,222
4.5×10^6
150 million
4.45×10^8

4
22
40
50
224
1454
1707

6
18
22.35
1.6×10^3
1967
16,100
1.61×10^4

25
30
2×10^3

Hints and Additional Information

- Assume the average man weighs 180 lbs.
- Decimals are generally not used to represent a "count", or number of something (e.g., there is unlikely to be a fraction of a window on the Sears Tower).
- The population of the Chicago Metropolitan area (not including Indiana or Wisconsin regions) is 7.9 million. When estimating the number of men in Chicago remember to subtract the number of women and children from the total population of the Chicago metropolitan area.
- Michael Johnson runs the 200 meter and 400 meter dash races, but he certainly could not sustain the speed of 23.5 mph for a whole mile. The current fastest marathon runner, Khalid Kkannoucci from Kenya, runs 26 miles at an average speed of about 12.4 mph.

Appendix C

Solutions to The Gateway Arch: How High Can You Go, How Far Can You See?

Fill in the following paragraphs about the Gateway Arch with the appropriate quantities that complete the information. The numbers to be used in the paragraph are located in the list to the right:

The Gateway Arch, designed by Eero Saarinen in _1948_, was completed in 1965 and took 20_ months to complete. This catenary curve is 192_ meters wide and _630 feet tall, _85_ feet more than Mark McGuire's longest home run. On a clear day the visibility radius from the observation area is _30__ miles.

_3 X 10^5__ cu.ft. of earth and rock were excavated to prepare the site for the foundation. The Arch itself weighs _17,246__ tons, used 900_ tons of stainless steel, and cost _13 million_ dollars. Each year more than 10^6_ visitors take the tram ride to the top. The tram has been operating over _30_ years, traveled over _2.5 X 10^5__ miles total, and carried over 25 million_ passengers.

The observation platform has a maximum capacity of _100-140 visitors. The _40_ seat passenger train's speed is 340_ feet per minute, but the service elevator travels at a rate of 400_ feet per minute. The passenger's round trip takes 10_ minutes, and there are _1076_ steps in each leg from the base to the top for emergency purposes. The Arch was built in 142_ sections and, to ensure that the constructed legs would meet, the margin of error for failure was _1/64____ inch. In a _150_ mile per hour wind, the Arch will deflect _18_ inches at the top, but the usual sway is only _1/2_ inch.

The Gateway Arch is the tallest national monument in the United States and the 4th__ most visited tourist attraction in the world. The Eiffel Tower, however, is 354__ feet taller than the Arch, and the Sears Tower is 824__ feet taller. Egypt's Great Pyramid is 180_ feet shorter than the Arch; and both the Washington Monument, at _555_ feet tall, and the Statute of Liberty, _305_ feet tall, are shorter than the Arch.

20
30
85
192
630
1948
1965

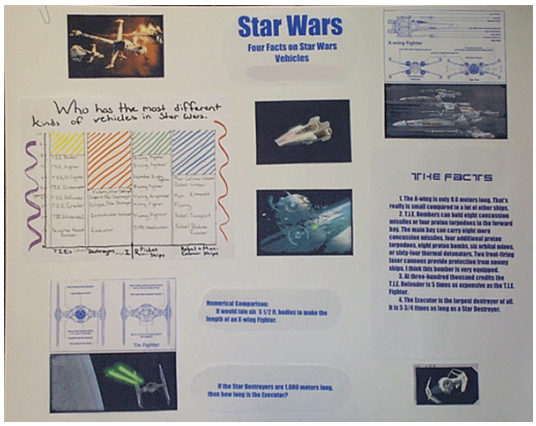
30
900
17,246
2.5×10^5
3×10^5
10^6
13 million
25 million

1/64
1/2
10
18
40
100-140
142
150
340
400
1076

4th
180
305
354
555
824

Appendix D

How Far, How Fast? How Many, How Much?
Theme Poster—it's your pick! (Complete Instructions)



Poster in photo by Mark Bernsen

What grabs your interest???

The goal of this project is to pick a theme or topic that interests you, and to explore it from a numerical perspective.

What is an appropriate topic? Almost anything that interests you:

- ***SPORTS? It could be Michael Jordan, your favorite college sports team, Olympic sport, a professional team, Or***
- ***MUSIC? What about a style of rock band? Or the great jazz trumpet players? Or Billboards Top 100? Or***
- ***NATURE? What about tigers or whales or even*** bugs***? Or…***
- ***MOVIES? The Star Wars series, or Indiana Jones adventures, or Independence Day? Or…***
  POLITICS? The 2000 Presidential Elections , or…
- ***ALMOST ANY TOPIC that interests YOU!!!***
  Don't limit yourself!

1. Choose a topic that interests YOU, and research it carefully.
   Try to find as much numerical information about the topic as you can.
2. Write six sentences that contain numerical facts related to your theme. Try to include as much variety as possible in the types of numerical information you use in your sentences.
3. Make a graph or chart. It can show relationships among your facts or it can show how one of your facts is related to facts of a similar theme. Be sure to include a title and appropriate labels for your graph.
4. Use a **additive comparison, or a comparison that uses addition/subtraction,** to make one of your sentence facts more meaningful. Use phrases like "larger than" and "less than" to describe the comparison. Include a visual representation of that comparison.
5. Use a **ratio** to illustrate or describe a comparison for a different numerical fact. Use words like "times as many", "stacked on top of", or "laid end-to-end" to describe the comparison. Include a visual representation of that ratio.
6. Write three questions that could be answered after analyzing the information that you have provided. Ask at least one question that requires looking for a pattern or trend suggested by the graph. Include the answers, but hide them.
7. Compile all of the above into a poster presentation that is visually interesting and appealing. Neatness and creativity count!

Appendix E

Sample Assessment Tool for Poster Activity

Name _____

THEME POSTER ASSESSMENT

_____ Six numerical facts are presented. Complete sentences with proper punctuation, capitalization and spelling are used. (30)

_____ An appropriate graph or table is used to illustrate relationships. The graph has a title, labels and correct scales. (15)

_____ An additive comparison is used to clarify a numerical fact. (10)

_____ A ratio is used to illustrate or describe a comparison for a different numerical fact. (10)

_____ The poster poses 3 questions that require analysis of the information presented. (15)

_____ The information is presented on a poster that is attractive and neat. (15)

_____ The poster has a title and the author's name is visible on the front. (5)

_____ The poster goes beyond the minimum requirements.

TOTAL _____

Appendix F

How Big Is the Base of the Great Pyramid of Giza? Geometer's Sketchpad Directions

The Great Pyramid of Giza in Egypt has a square base that is 755 feet on each side. To help us think about how big this really is we can compare it to a square that we can measure in the classroom. Use masking tape to enclose a square on the floor of the classroom that is 10 feet on each side. How many times will that square fit inside the base of the Great Pyramid?

Let's use Geometer's Sketchpad to make a scaled drawing of the comparison that helps us think about the size of the base of the Great Pyramid of Giza.

1. Open **Geometer's Sketchpad**. Make the page as large as possible.
2. Go to the **Display** menu and choose **Preferences**. Change the **Distance unit** to **Pixels**.
3. Click on the **OK** button.
4. Go to the **Graph** menu and choose **Show grid**.
5. Go to the **Graph** menu again and choose **Hide axes**.
6. Draw a square that is 10 pixels on a side. This will be a scale drawing of the square that has been outlined with tape on the classroom floor.

    - Use the **segment tool** to draw a segment that is 10 pixels long. (The grid on the sketch-pad page has dots that are 10 pixels apart.)
    - Select the line segment and go to the **Measure** menu and choose **Length**. Sketchpad will print the length of the line in pixels in the upper left-hand corner of the sketch.
    - Use segments of the same length to complete the square.

7. Draw another square that is 755 pixels on a side. This will be a scale drawing of the base of the Giza Pyramid.

(Note: to see the entire square, you will need a computer monitor with a pixel resolution greater than 775 pixels; in other words, a monitor with a large screen area)

Compare the small square to the large square. Remember that the small square is a scale drawing of the 10 foot square in our classroom and the large square is a scale drawing of the base of the Great Pyramid of Giza. How many times will that 10 foot square tile fit inside the base of the Great Pyramid of Giza? Explain your answer below.

The Sketchpad file can be found on the Web at <www.mste.uiuc.edu/m2t2/number/gizabase.gsp>

Appendix F (continued)

Additional Resources for Egyptian Pyramids

MacCaulay, David. *Pyramid*. Boston: Houghton Mifflin Company, 1975.

Stewart, Kelly. *Architecture*. Mountain View, California: Creative Publications, 1996.

Pyramid internet sites:

http://www.sdcoe.k12.ca.us/score/pyram/pyramtg.html

This supplemental unit to *Pyramid* was developed as part of the Schools of California Online Resources for Educators

http://www.pbs.org/wgbh/nova/pyramid/explore/khufuenter.html

http://www.newton.cam.ac.uk/egypt/

http://www.egyptvoyager.com/

Appendix G

Assessment Issues: Rubrics in General

Types of Assessment:

- ***Formative assessment:*** ***Informal assessment that occurs during the instructional period. As we teach, or as the students are doing an activity, how do we know if the students are understanding the intended concepts? Most often, this type of assessment comes from watching students progress and asking them questions in class as they work.***

- ***Summative assessment:*** ***More formal assessment that seeks to evaluate a student's work and mathematical understanding at the conclusion of a lesson, activity, or project. Most often, we associate summative assessment with grades, test scores, project grades, etc.***

- ***Tip:*** Although any "grade" before the "final" grade is a type of ***summative assessment***, all of those grades serve a dual purpose—they give information about a student's progress and understanding of topics covered. Thus, most "graded" work is also a type of ***formative assessment*** as well.

- A ***rubric*** is an instrument that serves as a guideline for evaluating student work. While there has been a great deal of attention paid to use of scoring rubrics in mathematics assessment, oftentimes, there is little discussion of exactly what a rubric is.

Holistic rubric

- A scoring scheme that focuses on product completion rather than process components.
- If we give "all or nothing" credit, it is, in essence, a holistic type of grade.
- A popular example of a holistic rubric is: ***0 points***—no effort and no work done; ***1 point***—student attempted to do the work and partially completed the assignment or it is partially correct; ***2 points***—assignment completed and all work is correctly done.
- This is often used when the effort is more important than the correctness of individual parts of an assignment. Example: One might only grade selected problems given in homework, but will give a holistic grade to signify if the homework assignment was attempted and/or completed.

Analytic rubric

- A scoring scheme that focuses on the process and components of an assignment more than its attempt and completion.
- The goal is to analyze a problem, project, or assignment to decide the relative importance of each part of the task and to categorize these elements. Points are then assigned according to the level of sophistication or correctness in each category.
- Sample categories for an analytic rubric are: ***correctness of mathematical procedures, strategies for setting up a problem or solving a problem, the use of appropriate representations, effective communication of one's understanding and strategies, etc.***
- ISBE uses a three-category analytic rubric for extended-response items on the ISAT: ***Mathematical knowledge (procedure, correct solution), strategic knowledge (reasonableness, problem-solving strategies), and explanation (communication, justification).***

For more information on this, see

Illinois State Board of Education: Assessment Section. (1995). Effective scoring rubrics: A guide to their development and use. Springfield, IL: Illinois State Board of Education.

Appendix G (continued)

Assessment Issues

Discussion Items for Teachers

1. In the Sears Tower Activity, what are the important mathematical concepts that arise?

2. If you were doing this activity with a group of students, would you want to give a grade for this activity?
   If so, what are the most important considerations in grading this activity?

3. Is correctness of a response more important than a student's rationale for choosing a particular response? Why or why not? How would you measure rationale? Is completion or discussion more important than correctness of the responses?

Troubleshooting Tips:

- ***If you are assigning credit for correct responses, how do you handle the fact that if any one response is incorrect, then at least one other response will also be incorrect?***

- ***Is there a difference between an incorrect response that is reasonable but just the wrong number versus an incorrect response that makes no sense whatsoever given the context or types of units involved?***

Part 1 of the Theme Poster asks participants to identify a topic of interest, or theme, and to research for numerical facts about that topic. The students are asked to construct 6 sentences that express the numerical information they have found and prepare these for inclusion on their poster.

4. How do you, as a teacher, evaluate this beginning portion of the poster-building? What understandings of number sense do the students need to exhibit? What understandings of sentence construction do the students need to exhibit?

5. This beginning activity seems like such a simple task!—Are you pleased with only basic information? What if I chose the theme of "Star Wars Movies" and one of my 6 sentences is "There have been 4 Star Wars Movies."? What is your reaction to this? Is your reaction individualized to the student? Why or why not?

6. Examine the required elements of the entire poster project. Having only started this project and module, what type of rubric do you think that you would use to evaluate this project? What concepts, skills, and understandings are necessary for the student to complete this poster? Are there further skills and understandings that would indicate an exceptional project effort?

Appendix G (continued)

Assessment Issues: Rubric Development

Below are several questions that could be helpful in assisting someone to create a rubric.

- What type of rubric would be best suited for assessing a project like the Theme Poster—holistic or analytic?

- On Day 1, the important consideration was ***completion*** (were there six sentences? Did each sentence contain a numerical fact about a common theme?). On Day 2, the focus has been on creating graphs to represent numerical information. What are the most important considerations for assessing graphs?

- What are the important parts of a graph?

- What is the relationship between the type of data and the type of graph used to represent that data? Are certain graphs more appropriate for certain types of data?

- Look at the requirements for the entire poster. What does each student need to be able to do in each of the following categories in order to make an acceptable poster or to make an exceptional poster?

  Mathematical skills or conceptual understandings:

  Problem-solving strategies, procedures, and reasoning:

  Presentation and explanation:

Appendix G (continued)

Assessment Issues: Making a Rubric for the Poster

1. What type of rubric would be the most appropriate for assessing ,or grading, the Theme Poster project? Explain.

2. Draw a sample graph in the space below that is similar to what you would want students to create for their poster. Identify all aspects of the graph that students should include to demonstrates their understanding of the data.

3. What categories would you include in an analytic rubric for grading the Theme Poster? Look back at all of the requirements for the poster. Write out your ideas for a rubric below and be prepared to discuss your decisions in tomorrow's workshop.

Appendix G (continued)

Assessment Issues: Making a Rubric for the Poster (Conclusion)

One issue of assessment is "self-evaluation" or "peer evaluation" by the students. In what ways could the students themselves participate in the evaluation process? Share your ideas in your group.

Use the space below to flesh out your ideas of a rubric appropriate to evaluating the Theme Poster project. Think about all of the components of the poster, and what mathematical understandings you would hope students exhibit through each component. Be sure to define the categories, the number of points for each category, and how much each component is worth in each category.

Remember that the number of points you assign to any component reflects the importance you place on that component. Of course, this information will also be of great interest to your students. Be prepared to justify your decisions.

Acknowledgements and Resources

Teaching requires daily use of the knowledge and skills gained through a lifetime of experiences. Ideas come from many sources. We share successful lesson ideas with each other and alter them to fit the students in our classroom. We are indebted to all of those who have taught us and taught before us for their inspiration and example.

The lessons included here were directly inspired by the following:

- Kelly, Brendan. *Authentic Learning Activities in Middle School Mathematics: Number and Operation*. Burlington, Ontario: Brendan Kelly Publishing, 2000.
- Reys, Barbara J. *Developing Number Sense in the Middle Grades: Addenda Series, Grades 5 - 8*. Reston, Virginia: National Council of Teachers of Mathematics, 1996.
- Rubenstein, Rheta N., Timothy V Craine, Thomas R. Butts and others. *Integrated Mathematics 1*. Evanston, Illinois: McDougal Littell A Houghton Mifflin Company, 1998.
- Venters, Diana and Elaine Krajenke Ellison. *Mathematical Quilts – No Sewing Required!* Emeryville, California: Key Curriculum Press, 1999.

Other resources:

- www.education.ti.com

This site offers support for educators and students. There are free classroom activities and downloads. There are also free online courses and tutorials.

- Erickson, Tim. *United We Solve*. http://www.eeps.com/
- http://mathforum.org/
- http://mathforum.org/workshops/sum98/participants/sanders/

3-D Drawing and Geometry by Cathi Sanders

- http://trackstar.4teachers.org/trackstar/ts/viewTrack.do?number=07124

Exploring Circles by Karen Hemmerling

- www.math.rice.edu/~lanius/

Informative math web site by Cynthia Lanius. One of these is the dueling pinwheels lesson for Geometer's Sketchpad. The Hand Squeeze is a data collection and analysis activity.

- www.csrnet.org/csrnet/substitute/math.html

A listing of math lesson plan sites

- www.parentech.org

Technology information initiative for families of middle school students A project created by NCREL and funded by Ameritech

- http://nces.ed.gov/nceskids/graphing/

Allows users to input data and choose from five types of graphs. The graphs can be printed or emailed.

- http://www.kidsclick.org/

Web search for kids by librarians.

- http://www.robertniles.com/data/

Suggestions for using the internet to find data

For more information about Mathematics Materials for Tomorrow's Teachers visit the website for the Office for Mathematics, Science and Technology Education.
www.mste.uiuc.edu

Number Sense