M2T2

Algebra

STATE GOAL 8:

Use algebraic and analytical methods to identify and describe patterns and relationships in data, solve problems and predict results.

Statement of Purpose

The NCTM Standards call for students to be able to "describe, extend, and make generalizations of geometric and numeric patterns." In this module we emphasize both patterns and making generalizations, building the foundation for more complex algebraic reasoning by having students represent algebraic relationships through graphs, tables, and charts. We encourage a hands-on approach that uses pattern blocks but also moves toward abstract reasoning, using number sentences and balances to reify the notion of equivalence. We want participants to look for patterns, to find different relationships, and to reason using patterns to make sense of mathematics problems.

We begin by looking at values that are assigned to objects and how these can be used to make predictions. Using the analogy of the balance, participants must find unknown values based on known values and relationships. Next they use a classic activity with pattern blocks and growth to observe and record patterns.

Connections to the Illinois Learning Standards.

Connections to the Illinois Learning Standards.

Standard 8.A.—Describe numerical relationships using variables and patterns. Throughout this module, numerical relationships are key. First, participants create number sentences that express relationships between values. Next, with the balance, they use relationships to solve problems. Finally, they use relationships to determine rules. This leads naturally to important algebraic reasoning and a foundation for the concept of functions.

Standard 8.B.—Interpret and describe numerical relationships using tables, graphs, and symbols. Tables are used to find growing patterns shown with pattern blocks. The data from the table is graphed and the relationships are described verbally and symbolically.

Standard 8.D.—Use algebraic concepts and procedures to represent and solve problems.

Table of Contents

Note: Appendices are printed only on the odd pages. This is done to make photocopying easier. That is, each participant should have a copy of all the odd numbered pages. While the instructors should have a copy of all the pages.

Materials

MATERIALS LIST

- Construction paper
- Glue
- Scissors
- Ribbon or paper tape
- Chart paper or poster board
- Pattern blocks
- Overhead pattern blocks
- Pattern block template
- Computer access
- Calculators
- Tape measures or meter sticks

Caterpillar Shape Sentences

Instructor Page

Make a bulletin board display of all the students' caterpillar Shape/ Number Sentences pages.

Reminder: within a single equation the value assigned to a given variable doesn't change. In the caterpillar below the red circle has a value of 3 each time it appears.

Values:
Red circle = 4
Green square = 2
Blue triangle = 5

Equivalent Shape Sentences

R + B + R + G + R + G
R + R + R + B + G + G
3R + B + 2G

Equivalent Number Sentences

4 + 5 + 4 + 2 + 4 + 2 = 18

4 + 4 + 4 + 2 + 2 + 5 = 18

5 + 2 + 2 + 4 + 4 + 4 = 18

3(4) + 2(2) + 5 = 18

Activity Instructions:

⇒ Before class the teacher cuts out several (each student will need a total of about eight) of each of the following shapes:
blue triangles red circles green squares

⇒ Each student to chooses a few of each shape.

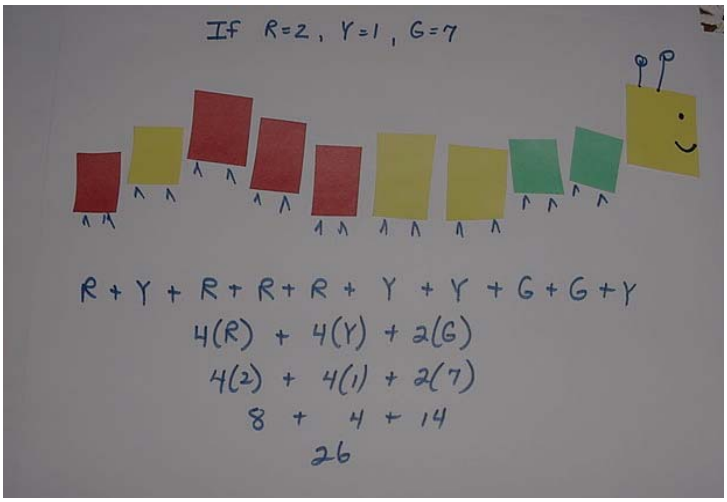
⇒ Each student places the shapes in a row to make a caterpillar, and then decorates the shapes with a head, eyes, antennae and other caterpillar appendages.

⇒ Each student then glues his/her caterpillar to the top half of a piece of plain paper to make a mini-poster. Also make a sketch of the caterpillar on the worksheet.

⇒ Next, the teacher announces the values of the three colored shapes, and each student writes a number phrase for his/her caterpillar.

⇒ Then each student writes several equivalent number sentences for his/her caterpillar on the worksheet.

⇒ Each student finishes the poster by choosing four of his/her number sentences and writing them below the caterpillar.



Discussion of Math Content

- Within a single equation the value assigned to a given variable doesn't change. Just as in a given story, the name "Max" always refers to the same character throughout the story. However, when reading a different story the same name "Max" will probably refer to a *different* character. The same is true with variables in mathematics. The value of a variable does not change within a problem, but when a different problem is started the value of the variable may change. For example, in one problem, X may be assigned the value of 3, but in a different problem X may be assigned a value of 13.
- The number sentences describing the Shape Caterpillar are equivalent forms of the same idea. They could demonstrate the Commutative and Associative Properties of Addition and the Distributive Property of Multiplication over Addition.

Participant Page

Caterpillar Shape Sentences

Sketch your caterpillar here

Equivalent Number Sentences

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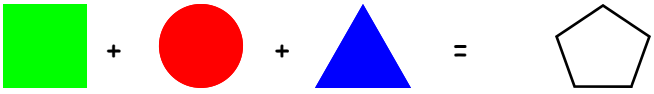
Instructor Page

Here we ask students to begin to make a distinction between the **value** assigned to a variable, and the **coefficient** of the variable. In these examples, the blue triangle is a variable that has been assigned the value 7. The coefficient of the variable is 2. It tells us that there are two triangles.

In both representations the value assigned to the hexagon is 23.

Shape Sentences

If the green square has a value of 2, the blue triangle has a value of 5, and the red circle has a value of 3, find the value of the pentagon.

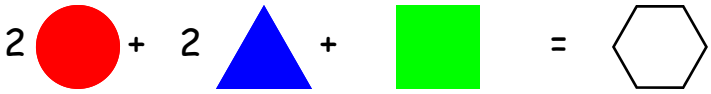


2 + 3 + 5 = 10

The value of the pentagon is 10.

If the green square has a value of 5, the blue triangle has a value of 7, and the red circle has a value of 2, find the value of the hexagon.

2 + 7 + 2 + 7 + 5 = 23



2(2) + 2(7) + 5 = 23

Using the values green square = 4, blue triangle = 9 and red circle = 2, the students should show a wide variety of representations of terms that total 72.

Discussion of Math Content:

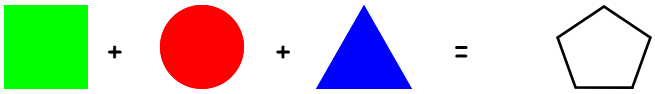
Some of the key ideas addressed in these lessons include:

- A given expression may have many different equivalent forms
- The order of terms in an addition expression can change without changing the value of the expression (Commutative Property of Addition)
- Like terms can be combined
- The same value can be represented in many ways.

Participant Page

Shape Sentences

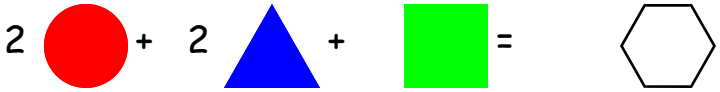
If the green square has a value of 2, the blue triangle has a value of 5, and the red circle has a value of 3, find the value of the pentagon.



If the green square has a value of 5, the blue triangle has a value of 7, and the red circle has a value of 2, find the value of the hexagon.

Does the value of the hexagon change if we change the order of the colored shapes? _____

Another, shorter, way to indicate two red circles is by using a number in front of the shape. Write a 2 in front of the red circle. The we can write:



If the green square has a value of 4, the blue triangle has a value of 9, and the red circle has a value of 2, make a shape sentence that has a value of 72. Show at least four different ways to do this.

M2T2

Mystery Shape Sentences

Instructor Page

In the previous lessons the **variable expressions** were on one side of the equation. Students were given values for the variables and then **evaluated** the expression. In this lesson variable shapes appear on both sides of the **equations**, and students are asked to **solve** the equations. They need number sense and knowledge of addition facts to find the missing value. At this time there is no formal instruction in methods for solving the equations. Students use their knowledge of arithmetic to solve the equations intuitively.

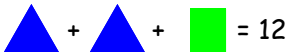
Find the value of a green square so that the sum of two green squares is 10. There are many pairs of numbers that total 10, but both green squares in a single equation have to be the same number and total 10. The only solution is 5. 5 + 5 = 10. The value of the green square is 5.

If the value of the red circle is 10, what is the value of the green square? _____

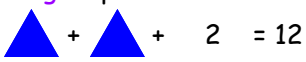
If the value of the red circle is 4 and the green square is 2, what is the value of the blue triangle ?

_____

If the value of the red circle is 4, then three red circles have a value of 12. Two blue triangles and one green square need to total 12.



If the value of the green square is 2. Two blue triangles plus 2 need to total 12.



Two blue triangles need to total 10.

The only solution is 5.

2(5) + 2 = 3(4)

Discussion of Math Content:

- Variables change their values in different situations, but keep the same value in a particular situation.
- Writing an equivalent form for an expression may make finding a solution easier.

Solutions for page D-9

5 + 5 = 10

2(5) = 3 + 7

2(5) + 2 = 3(4)

Extension:

Online Resource:

Use this applet to find mystery weights using a balance

http://www.mste.uiuc.edu/wofford/balance.html

M2T2

Participant Page

Mystery Shape Sentences

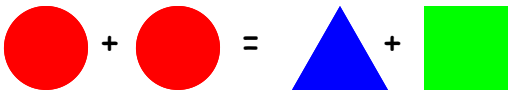
Hints or notes:

A numeral written next to a shape tells how many of that shape there are. It is called the **coefficient** of the variable.

The values of some of the shapes are given. find the value of the mystery shape.

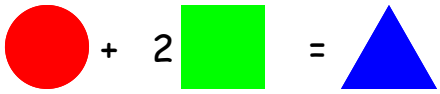
If the value of the red circle is 10, what is the value of the green square? _____

If the value of the red circle is 5 and the value blue triangle is 3, what is the value of the green square? _____

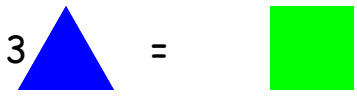


If the value of the red circle is 4 and the green square is 2, what is the value of the blue triangle ? _____

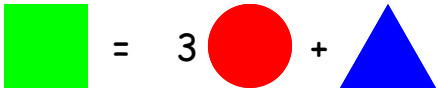
Find the values of the mystery shapes. If there is more than one possible value, give them all.



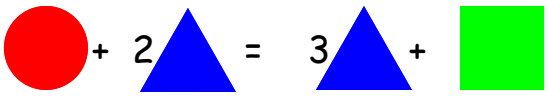
blue triangle = 7 red circle = 3 green square = _____



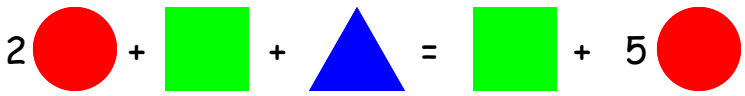
blue triangle = _____ green square = 12



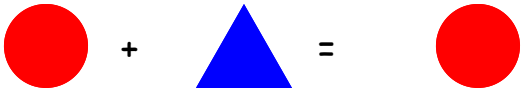
blue triangle = 4 red circle = _____ green square = 10



blue triangle = _____ red circle = 5 green square = 2



blue triangle = 9 red circle = _____
green square = 3

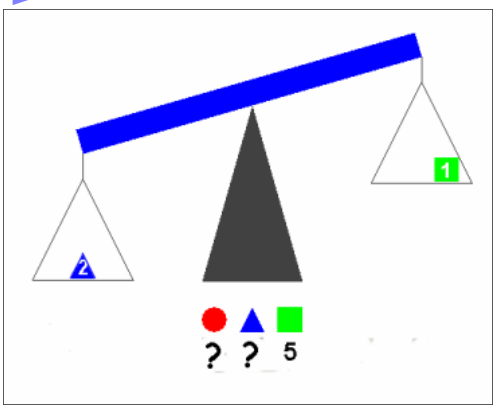


blue triangle = _____ red circle = _____

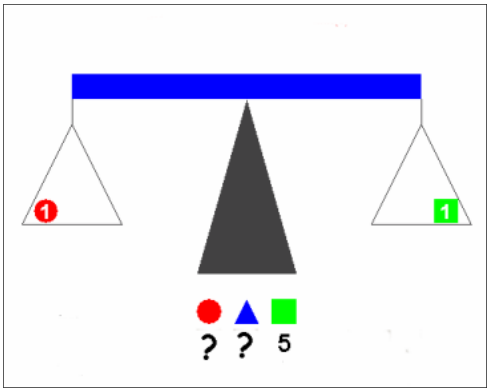
Weighing In

Instructor Page

The series of three pictured situations give clues for one solution for each of the three shapes.

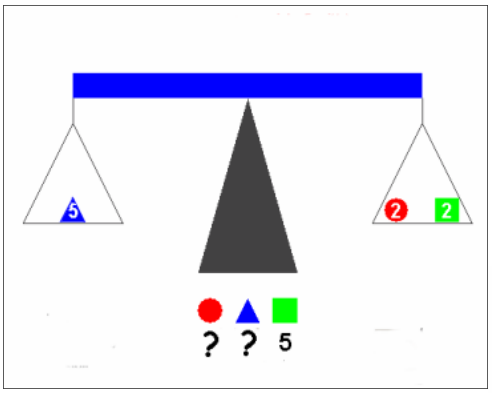


- Each square weighs 5 units.
- There is one square on the balance.
- There are two triangles on the balance.
- Two triangles are heavier than one square.



- One circle balances one square

From this clue we know that the value of the circle is 5 units.



- Five triangles will balance two circles and two squares.

Use the information in all three pictures to determine the value of the objects.

From this clue we know that the value of the five triangles is 20 units, so one triangle must be 4 units

■	5
▲	4
●	5

Notice that this problem could have been solved without using some of the information in the first balance.

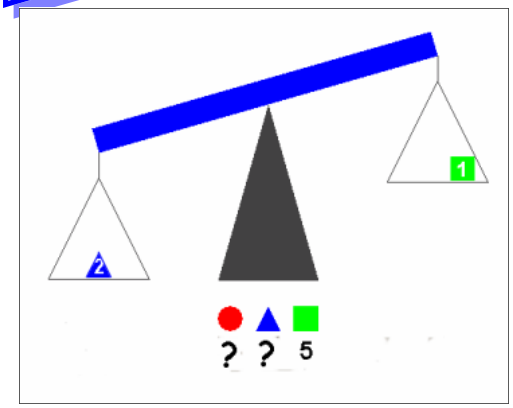
This activity is meant to be used with the online applet at www.mste.uiuc.edu/wofford/balance/. The applet is also available on the M2T2 CD.

Appendix A has a student paper and pencil activity

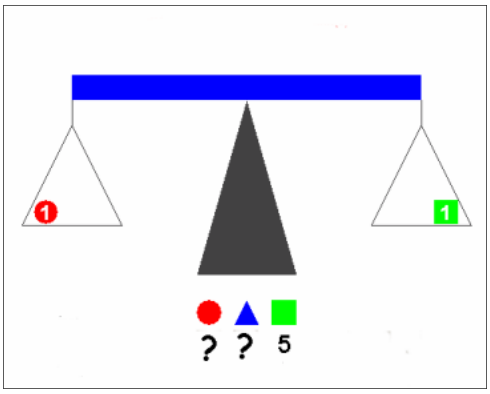
Participant Page

Weighing In

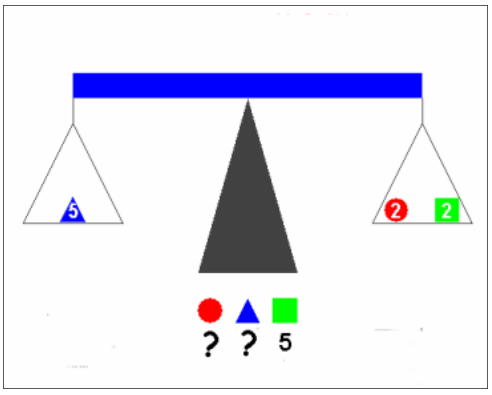
Determine the weight of the objects from the information in the pictures. The values for the weights can only be whole numbers from 1 to 6.



- Each square weighs 5 units.
- There is one square on the balance.
- There are two triangles on the balance.
- The two triangles are heavier than one square.

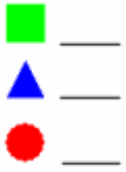


- One circle balances one square



- Five triangles will balance two circles and two squares.

Use the information in the pictures to determine the values of the objects. Enter those values beside the pictures below.



Number Tricks

Instructor Page

Introduce the lesson by proclaiming that you can guess the last digit of anyone's telephone number. All you need is a few computations. Choose a student to help with the trick by doing some arithmetic.

Tell the student to write the last digit of his/her phone number on a small piece of paper and hand it to another student for safe-keeping. Then follow these instructions:

1. Write the last digit of your phone number on a piece of paper and give it to a classmate.
2. Add two.
3. Multiply by three.
4. Subtract four.
5. Add one more than the number you chose.
6. Add five.
7. Divide by four.
8. Announce your result.

When the student has completed the calculations listed, ask for the result. Subtract 2 and tell the class the last number of the student's phone number. Perform the trick again with a few more students.

How did you do it? Can you teach us how to do it? How does it work?

We can explain "how it works" using algebra. First use a box symbol for "the number", and ◊ for one. The second column of the chart at the right shows how to represent the calculations symbolically. Everytime the steps are followed correctly the result is two more than the starting number. The third column shows how to represent the steps using a variable for the starting number

Verbal Instructions	Use symbols to show the steps	Show the steps algebraically
Choose a number. Write it here	□	x
Add two	□ ◊◊	x + 2
Multiply by three	□ ◊◊ □ ◊◊ □ ◊◊	3(x + 2) or 3x + 6
Subtract four	□ ◊◊ □ □	3x + 2
Add one more than the number you chose.	□ ◊◊ □ □ ◊ □	4x + 3
Add five	□ □ □ □ ◊◊◊ ◊◊◊◊◊	4x + 8
Divide by four.	□ ◊◊	x + 2
Write your result here	Your number + 2	x + 2

Online Resources:

Lanius, Cynthia. Algebra - Fun with Calendars. http://math.rice.edu/~lanius/Lessons/calen.html

Richard Briston's Math Assignments: Number Tricks. http://mathforum.org/briston/briston.tricks.html

Moss, Larry. Teaching Magic As a Math Topic. http://www.fooledya.com/moss/papers/mathfun.html

Participant Page

Investigating the Number Trick

If you can follow the steps listed here, I can guess the last digit of your phone number. Don't tell me what the number is, and don't tell what you get after any of the steps until you get to the last one. If you figure out my trick, don't tell until everyone has had a chance to think about it.

1. Write the last digit of your phone number here. _____
2. Add two. _____
3. Multiply by three. _____
4. Subtract four. _____
5. Add one more than the number you chose. _____
6. Add five. _____
7. Divide by four. _____
8. Write your result here. _____
9. Tell me what you wrote for step 8 and I can tell you the last digit of your phone number.

We can use algebra to figure out how this trick works. To help analyze the number trick, use □ to stand for the number you choose. Use ◊ to stand for one. Then □◊◊◊ would be a symbol for "your number plus three". Repeat the steps of the number trick using these symbols to show each operation.

Next, use algebraic notation to show the steps. In the place of the □ use **x**. Then □□ could be represented by **2x**, and **2x** + 6 would represent □□◊◊◊◊◊◊.

Verbal Instructions	**Use symbols to show the steps**	**Show the steps algebraically**
Choose a number. Write it here		
Add two		
Multiply by three		
Subtract four		
Add one more than the number you chose.		
Add five		
Divide by four.		
Write your result here		

Growing Linear Patterns

Instructor Page

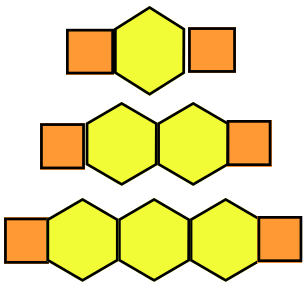
In the early grade levels many of the patterns that teachers and students work with are **repeating** patterns. Students may have almost daily experiences with patterns such as "Snap, snap, clap, snap, snap, clap" or "Blue, red, red, blue, red, red" or "ABAB". The children are accustomed to predicting what comes next, describing the patterns (verbally, with pictures or models) and finding the missing component in the pattern.

This lesson and those that follow will expose students to growing patterns. They will build their growing pattern with pattern blocks, then they will extend their pattern, describe it in words, describe it with a graph and represent it

Record the critter growing activity on chart paper as each student records on his/her own paper.

A recording sheet can be found in the appendix

The lesson begins with the teacher displaying on the overhead "a one-day old critter, a two-day old critter and a three-day old critter" made with orange square pattern blocks and yellow hexagons pattern blocks. See illustration.



One day old critter

Two day old critter

Three day old critter

Students are then asked to make a four day old critter with pattern blocks. The teacher asks one student to describe his/her four-day old critter. Next another student is asked to give directions for making a five-day old critter. The teacher then asks the students, "how many blocks will it take to make a ten-day old critter?" Then the teacher asks, "how many blocks would it take to make a hundred-day old critter?" Most students can answer 102. Ask how they know the number of blocks without building all of the critters. Ask them to give you a rule for figuring the number of blocks if someone tells them how many days old the critter is. They will say something like, "there are always 2 orange blocks and there is one hexagon for everyday so you just take the number of days and add two to it and that will be the number of blocks." Write the student's rule in words on chart paper that you can refer to in the next lesson.

Point out the power of finding the pattern. It allows us to know the number of blocks without having to build each critter.

The students should build the critter for each of the first five days and record the number of the day and the number of blocks in the chart on the participant page. There is a page in the appendix that students can use for drawing their critters and recording the pattern.

In the second lesson you will show the students how to rewrite their rule using an algebraic expression.

Extension

What percent of your critter is yellow on day 48?
What percent of your critter will be yellow on day 98? How did you figure this out?

"To think algebraically, one must be able to understand patterns, relations, and functions; represent and analyze mathematical situations and structures using algebraic symbols; use mathematical models to represent and understand quantitative relationships; and analyze change in various contexts. Each of these basic components evolves as students grow and mature." - NCTM, *Navigating Through Algebra in Grades 3-5*, p. 2.

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Participant Page

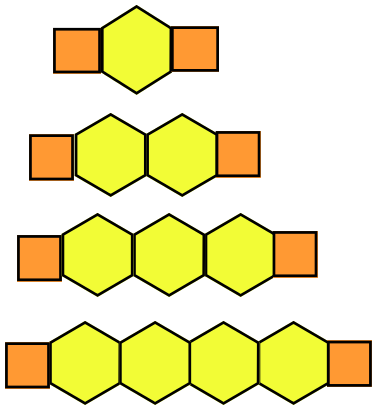
Growing Linear Patterns

Use your pattern blocks to build a critter at each age. Record the total number of blocks used to build the critter at each age.

Journal Activity:

Describe the pattern.
Tell what stays the same in the pattern?
What changes?

If someone tells you how old the critter is, how would you figure out what the critter looks like?



Number of days old	Number of blocks
1	3
2	4
3	5
4	6
5	
6	
10	
100	

Show a critter that is five days old with your pattern blocks. Draw it in the space below. Fill in the number of blocks needed to build it in the chart. Complete the chart for the listed days. You may build the critters if you need to.

Graphing Linear Patterns

Instructor Page

Students have now made geometric models of their growing pattern with pattern blocks and then made a drawing of those models. That has allowed them to think about their patterns geometrically. They filled in charts to look at the pattern numerically. Then they described their patterns verbally and in writing using words.

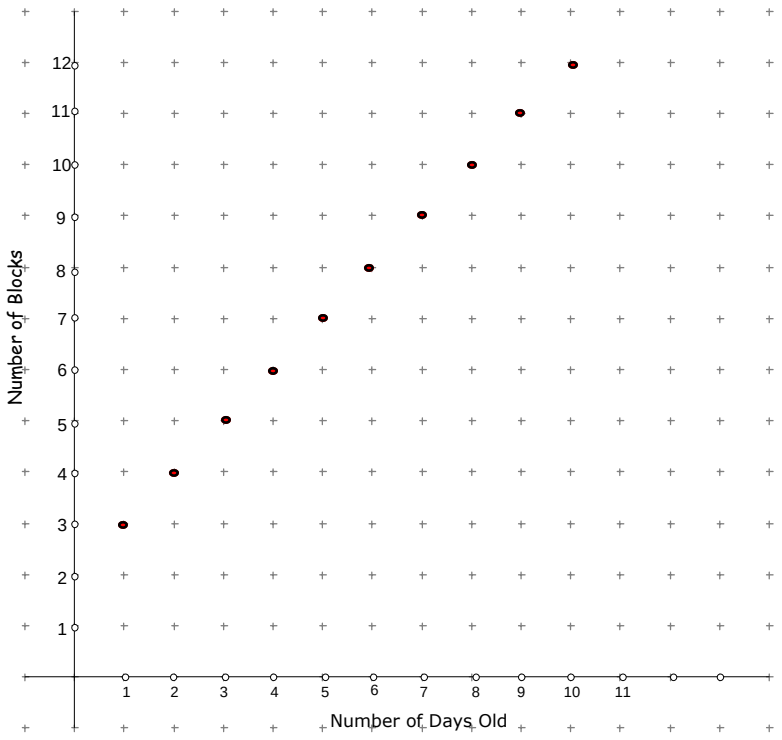
Graphs of ordered pairs may suggest a relationship. If the points can be lined up with a ruler or the edge of an index card (or any straight edge), the relationship is called **linear**.
We may also describe a relationship as **increasing** or **decreasing**. If the graph is linear, it increases or decreases at a **constant rate**.

This graph is linear. It increases by one block every time the critter gets one day older.

During this lesson students look at another way to describe their pattern – graphically. Make an overhead sheet of the blank graph (See the appendix for another blank graph) and use it to show how to plot the ordered pairs from the chart. Point out that where the axes (two number lines) cross is called the "origin". Point to the first ordered pair of numbers in your critter chart, (1, 3). This first coordinate (for the age of the critter) tells how many to count over on the horizontal axis. Begin at the origin and count over 1. The second coordinate (for the number of blocks) tells how many to count up. Count up 3 and place a point at the intersection. Proceed to do this with each ordered pair.

To help students remember how to graph ordered pairs it may help to compare graphing an ordered pair to walking into a building and taking an elevator up or down. In the ordered pair (3,2), for example, the three tells you how far you have to walk into a building to get to the elevator. The 2 tells you to go up 2 in the elevator. You can't take an elevator up or down until you walk into the building. (3, -4) Tells you to walk into the building 4 paces, then take the elevator down 4. (The use of this mnemonic is limited to values for the x-coordinate that are positive.)

Number of days old	Number of blocks
1	3
2	4
3	5
4	6
5	7
6	8
7	9
8	10
10	12
100	102



Online Resources:

Whitcher, Ursula. Chameleon Graphing: An Introduction to the Coordinate Plane. http://mathforum.org/cgraph/

M2T2

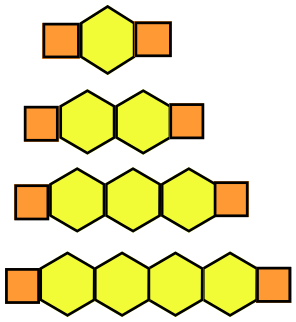
Participant Page

Graphing Linear Patterns

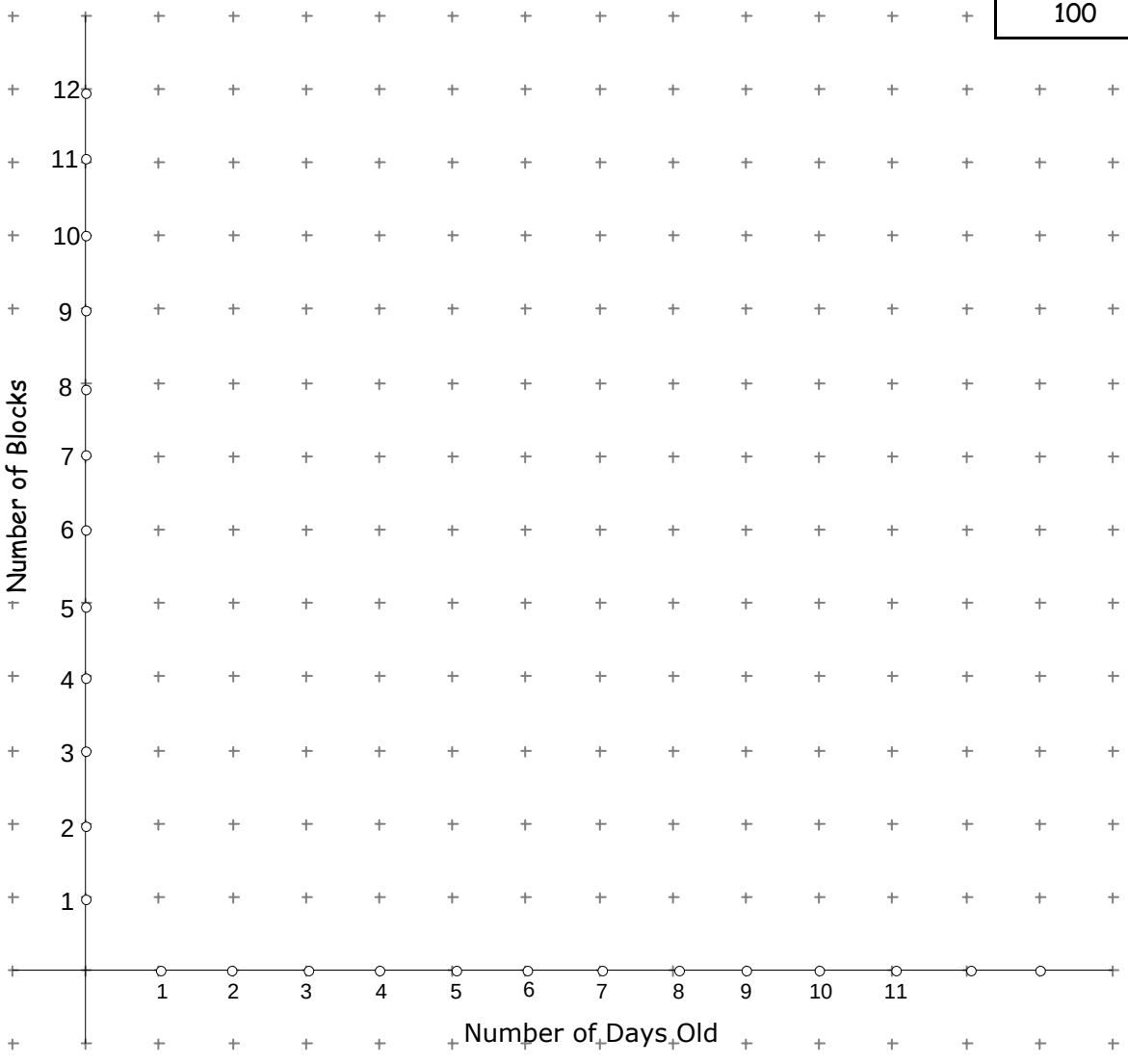
Journal Activity:

What does the graph show?
Describe the graph. Include answers to these questions in your description. Is it linear or nonlinear? Is it increasing or decreasing?

Graph the coordinate points from this same table to show the relationship between the age of the critter and the number of blocks.



Number of days old	Number of blocks
1	3
2	4
3	5
4	6
5	
6	
10	
100	



M2T2

Growing More Linear Patterns

Instructor Page

Students may have other methods for finding a rule for a pattern. Sometimes a student will first notice that the number of blocks increases each day by two. The pattern can be extended by counting by two's in the right column for the number of days increased in the right column. For example, at age 3 the creature has 8 blocks. To find the number of blocks in a creature of age 12, notice that there are 9 days between age 3 and 12 so count by two's 9 times starting from 8 blocks to get 26 blocks for the 9 day old creature. This description can be expressed symbolically if we remember that repeatedly adding 2 for the number of days is the same as multiplying the number of days by 2. **Blocks = 2 times days** needs to be adjusted to make it work so add 2. **Blocks = 2 times days + 2**

Context:

Apply the procedures used to describe and analyze the orange and yellow critter to this green and red creature.

Activity Instructions

⇒ Examine the one day old creature. Use pattern blocks to build it. Count the number of blocks needed to build it and record in the chart.

⇒ Build the two day old creature. Count and record the number of blocks needed.

⇒ Build the three day old creature. Build creatures that are four and five days old. For each, count the number of blocks and record in the chart.

⇒ Continue to fill in information about the number of blocks needed for creatures that are six, ten, and one hundred days old. Use your pattern blocks to build the creatures if you want to.

⇒ Discuss the methods suggested by the students for finding the number of blocks needed for a creature that is 100 days old.

⇒ Write a rule. There should be agreement that at every age the creature has 2 green triangles. Each day the creature "grows" two more red trapezoids. So the rule is something like, "start with 2 triangles and each day add two more red trapezoids."

⇒ Use symbols instead of some of the words:

Blocks = 2 triangles + 2 trapezoids times the age in days

⇒ Write the rule in algebraic form

$b = 2 + 2d$

⇒ Check the rule for 7 days and 100 days. Be sure it works.

⇒ Try to find another way to write the rule. Some possibilities are:

$b = 2(1 + d)$

$b = d + d + 2$

$b - 2 = d + d$

Resources:

Creating, Describing, and Analyzing Patterns. http://standards.nctm.org/document/eeexamples/chap4/4.1/Part2.htm

Cuevas, Gilbert J. and Yeatts, Karol. *Navigating through Algebra in Grades 3-5.* NCTM, Reston, VA, 2001.

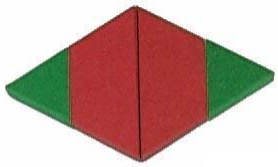
Moses, Barbara. *Algebraic Thinking, Grades K-12: Readings from NCTM's School-Gased Journals and Other Publications*. Reston, VA, 1999.

M2T2

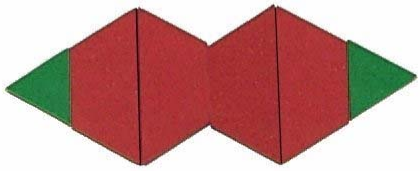
Participant Page

Growing More Linear Patterns

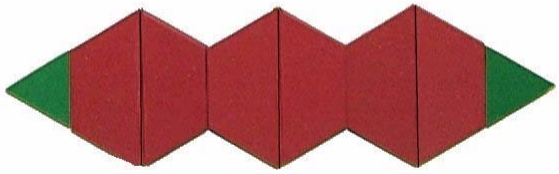
Use your pattern blocks to build a creature at ages from one to five days. Record the total number of blocks used to build the creature at each age listed in the chart.



One day old creature



Two day old creature



Three day old creature

Number of days old	Number of blocks
1	
2	
3	
4	
5	
6	
7	
10	
100	

What stays the same in your pattern? _____

_____

What changes?_____

_____

How do you know the number of blocks without building all of the creatures? Write a rule for figuring out the number of blocks if someone tells how many days old the critter is. _____

_____

Show that your rule works for at least two of your ordered pairs.

Try to find another way to write your rule. _____

Show that this form of the rule works for at least two of your ordered pairs.

M2T2

Graphing Another Linear Pattern

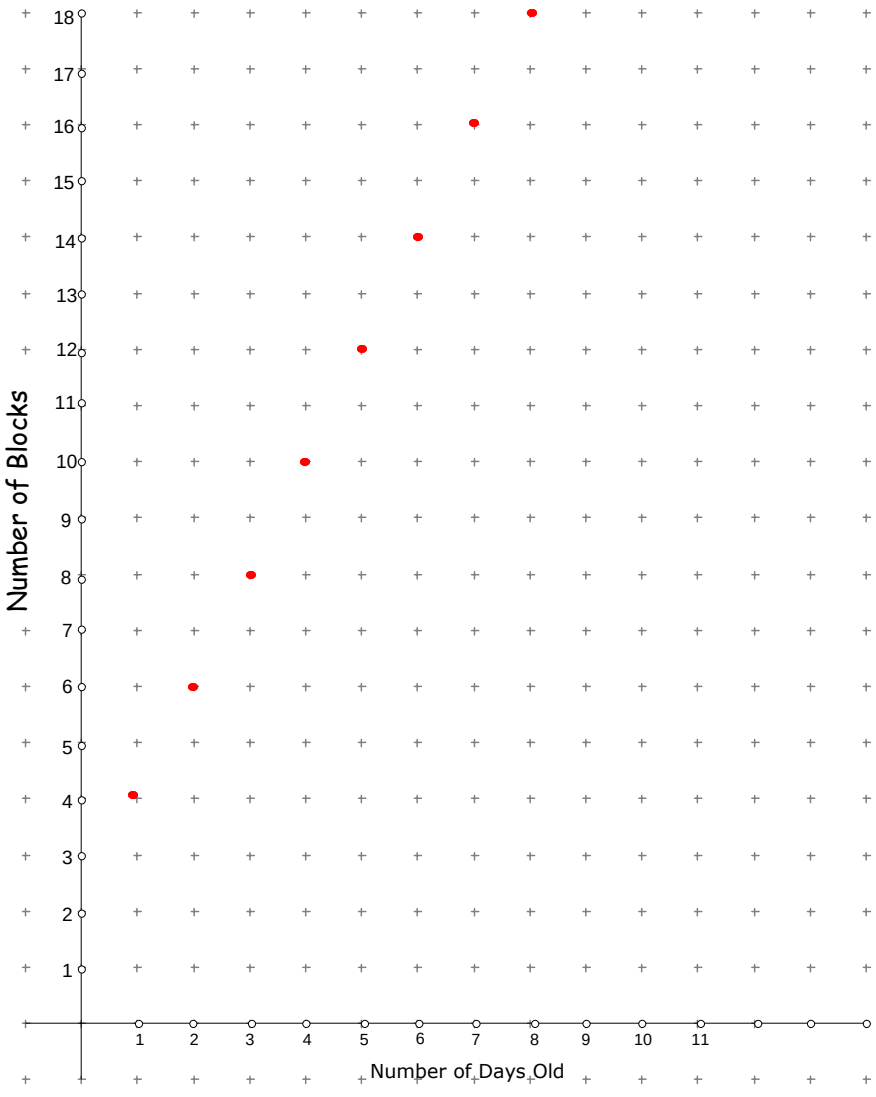
Instructor Page

It should be noted that because these growing patterns were built with physical materials, there are only whole numbers on the graphs. There are no fractions that come up and no zero. There are only non-connected dots on the graph. It seems clear that the dots lie in a straight line but technically speaking they cannot be connected because they are describing blocks that cannot be cut into pieces. The points on the graph are discrete points, that is, separate and not connected.

⇒ During this lesson students will look at the graph of a different linear pattern and compare it to the graph of the first pattern.

⇒ The red and green creature grows two blocks each day. Its rate of change is 2. The orange and yellow critter grows one block each day so its rate of change is 1. This makes the creature graph steeper than the critter graph.

⇒ In the critter pattern every day the critter has 2 squares plus the number of growing hexagons. In the creature pattern every day the creature has two triangles plus the number of growing trapezoids. Is this information shown in the graph?



This graph is linear and increasing. It increases by two blocks every time the creature gets one day older.

Number of days	Number of
1	4
2	6
3	8
4	10
5	12
6	14
7	16
8	18
10	22
100	202

M2T2

Participant Page

Graphing Another Linear Pattern

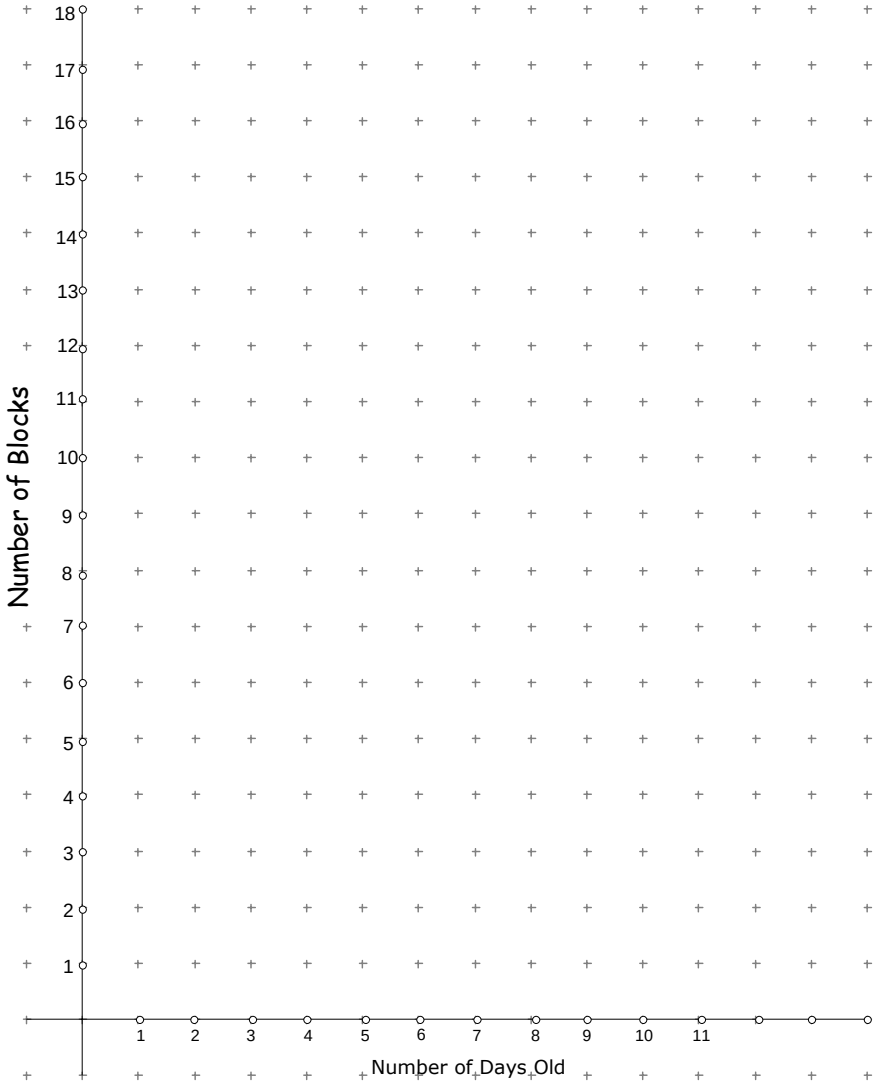
Journal Activity:

What does the graph show? Include answers to these questions in your description. Is it linear or nonlinear? Is it increasing or decreasing? How is it like the critter graph? How is it different?

Complete this table with the information from the creature activity in the previous lesson. Graph the coordinate points from the table showing the relationship between the age of the creature and the number of blocks.

Write your rule for this relationship here.

Number of days old	Number of blocks
1	
2	
3	
4	
5	
6	
10	
100	



M2T2

Instructor Page

Caution:

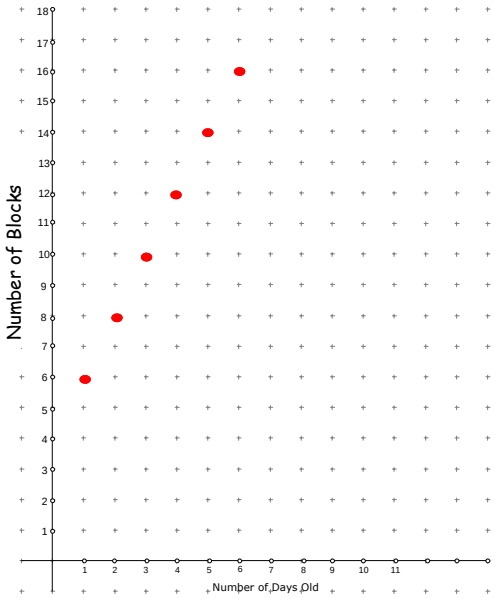
Students often begin making a pattern (especially with rhombii) that becomes circular. These will not work because they reach a point where the pattern can no longer grow. It grows back into itself. Another pitfall is a pattern that may grow in a zig-zag way. Like an up/down trapezoid pattern. These too, can be a problem when it comes to making a general rule for their pattern. When students generalize this kind of pattern they have to say things such as "for even numbers you do one thing and for odd numbers you do another."

Graphing and Describing Patterns
Making a Poster

Part 1

- Complete the chart and graph for the pattern shown.
- Discuss how the graph of this pattern compares to the previous graphs. Same steepness as the red and green creature. Steeper than the orange and yellow critter. Starts above the red and green creature
- What stays the same for each day of the pattern? Each day there are four brown rhombii
- What changes each day? The number of green triangles increases each day
- Write some of the verbal and algebraic descriptions of the pattern for the whole class to see and discuss. b = 4 + 2d is one possibility

Number of days	Number of blocks
1	6
2	8
3	10
4	12
5	14
6	16
10	24
100	204



Part 2

- Assign students to work with a partner and use the pattern blocks to make their own growing pattern. See the appendix to find a variety of sheets for recording.
- Use a chart to record the ordered pairs that describe the pattern.
- Graph the ordered pairs and describe the graph.
- Describe the pattern with words.
- Describe the pattern algebraically.

In the patterns that we've looked at so far we've kept track of age and the number of blocks. In their original pattern students may want to count something else. If they are going to build ships in a race (red trapezoids with green sails) they could keep track of the number of boats and the number of sails. They may want to make Christmas trees with an orange stem and a trapezoid body and a green triangle top. The tree would keep growing taller by one trapezoid. One student may make a daisy chain with orange squares for centers surrounded by yellow hexagon petals. That student could count flower centers to the number of petals. (One center, four petals; two centers, eight petals).

The pattern needs to grow in only one direction. The pattern needs to grow in a straight line horizontally or vertically, but not both. That limits the patterns to linear functions.

M2T2

Graphing and Describing Patterns

Participant Page

Draw your pattern for the first four days. Complete the table. Graph the coordinate points from the table. Use words to describe a rule for finding the number of blocks for a pattern of any age. Write an algebraic equation for the relationship between the age of the pattern and the number of blocks.

One day old pattern

Two day old pattern

Three day old pattern

Four day old pattern

Number of days old	Number of blocks
1	
2	
3	
4	
5	
6	
10	
100	

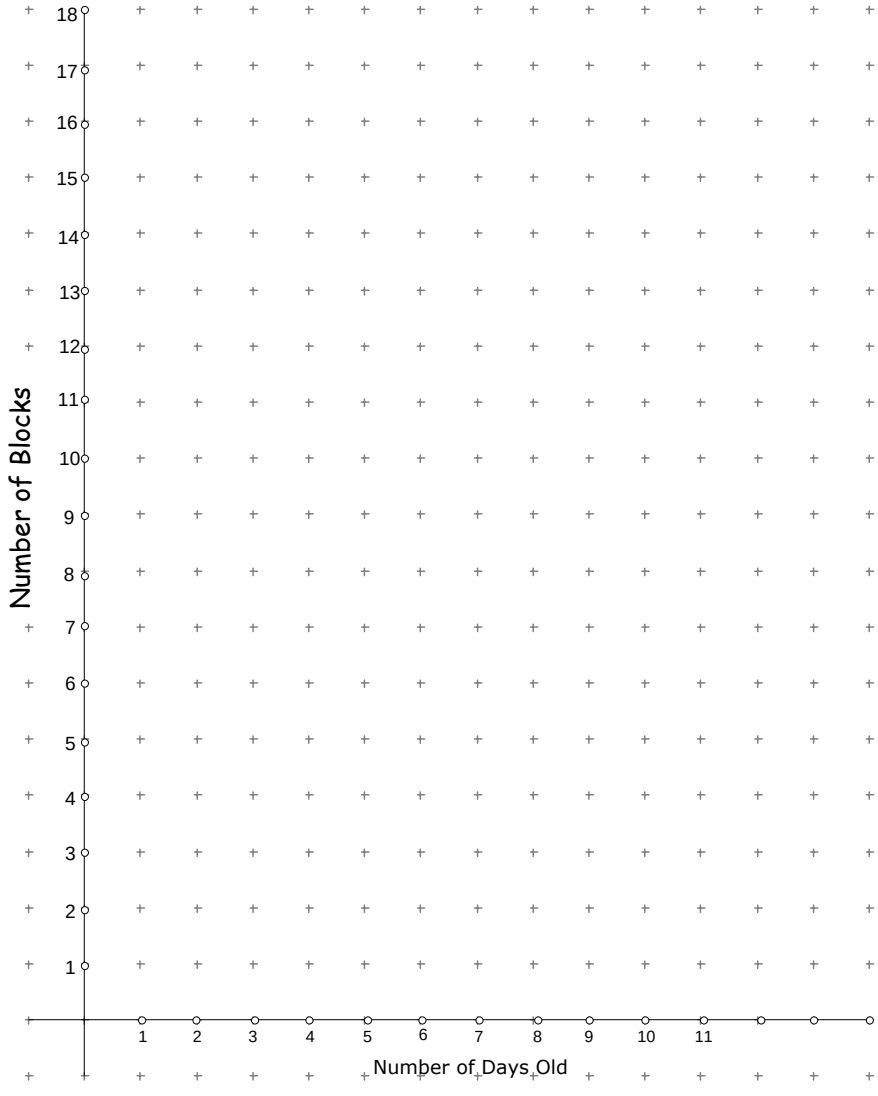
Describe the pattern. _____

_____

_____

_____

Write an algebraic equation describing the relationship between the age of the pattern and the number of blocks _____



Instructions for Constructing a Poster of Patterns

Instructor Page

Each pair of students chooses two of their growing patterns to display on one large piece of chart paper or poster paper.

For **each** pattern the completed poster should have:

- pictures of the pattern block models for at least the first four stages
- the completed chart
- the description of the pattern in words
- two or more equivalent algebraic expressions
- the graph
- a description of the graph

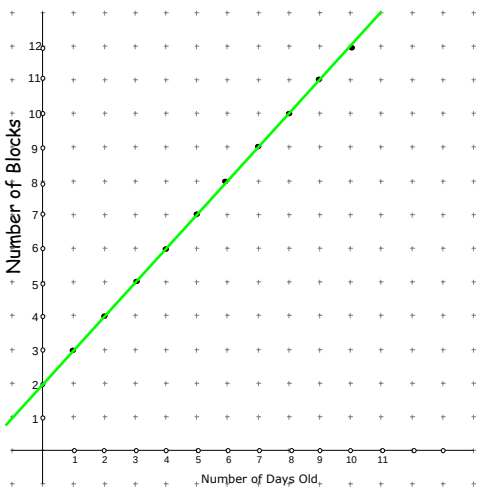
Write about how these two patterns are alike and how they are different.

Discussion of the Posters of Patterns

Discussing and processing this activity allows students to form general rules based on the work of everyone in the class. Each pair of students holds up their poster and describes some of the components. As students look at the patterns, ask what was the same in each picture. For instance, in the critter picture the orange square head and tail stayed the same in each picture. Point out that this is what mathematicians call a **constant**. Ask what changes in each picture (like the length of the critter or the number of yellow hexagons). Tell students that is what mathematicians call a **variable** because that varies or changes each time. Look at the algebraic expression and see that the **+2** is the same each time (so it's a **constant**) and the "d" (for days old) changes so it's a **variable**. The number of blocks changes so the "b" is also a variable.

As students look at the algebraic expression describing growth of the pattern being discussed, ask if anyone else has the same algebraic expression. Often there is someone. There will probably be entirely different pictures of growing models, each described by the same algebraic expression. Point out that an advantage of writing about a pattern using algebra is that it is very short and quick to write, but you lose some of the description or picture.

The students who have growing patterns with the same algebraic expression may stand up and hold their respective posters next to each other. Then the class can compare the two posters. The graphs, algebraic expressions, and charts will be identical but the written descriptions and pictures could be very different.



Place a piece of spaghetti over the points on the graph. The point where the spaghetti (or extended line) crosses the y-axis. (vertical axis) is called the **y-intercept**. In the critter model the **y-intercept** of the extended line is "2". It can be seen in the algebraic expression as the "2" in d + 2 and in the pattern block model as the 2 constant pieces of the orange head and tail. The growth of the critter starts on day one and jumps forward each day so the extended line and the algebraic expression only make sense for integer values beginning with one. Mathematicians would say that the **domain** (possible values for the days) is restricted to integers beginning with one and the relationship is **discrete** rather than **continuous**.

The **slope** of a graph is a measure of its steepness. The critter graph has a **slope of 1**, because each day the number of blocks increases one. The creature graph has a **slope of two**, because each day the number of blocks increases two.

M2T2

Constructing a Poster of Patterns

Participant Page

Work with a partner to make a poster to display two of your growing patterns.

For each pattern the completed poster should have:

- pictures of the pattern block models for at least the first four stages
- the completed chart
- a description of the pattern in words. (Tell what stays the same at each stage and what changes.)
- an algebraic expression
- another equivalent algebraic expression
- the graph
- a description of the graph

Write about how these two patterns are alike and how they are different.

Exploring Another Kind of Pattern

Instructor Page

If you start with 15 inches of ribbon and cut it in half, the first piece is 7.5 inches long. The next cut piece is 3.75 inches long. When it is taped to the first piece the new length is 11.25 inches. Since the original ribbon was 15 inches, the total of the pieces increasingly gets closer to 15 inches, but can never reach 15. This makes 15 the **limit** of the lengths of the ribbon.

We could represent this relationship algebraically using an infinite sum. These symbols describe what the students did. $\frac{1}{2}(15) + \frac{1}{2}\cdot\frac{1}{2}(15) + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}(15) + \ldots$ Mathematicians have a shorter way to write this. They use Σ to mean "sum" and write

$$\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n 15$$

The growing patterns that we produced in the previous lessons have all been linear relationships. Each time the pattern blocks increased by a constant value. There was a constant rate of change.

In this ribbon cutting activity, the ribbon length increases by half of itself each time. The amount of increase becomes smaller and smaller. This is a nonlinear relationship. The amount of increase is not constant.

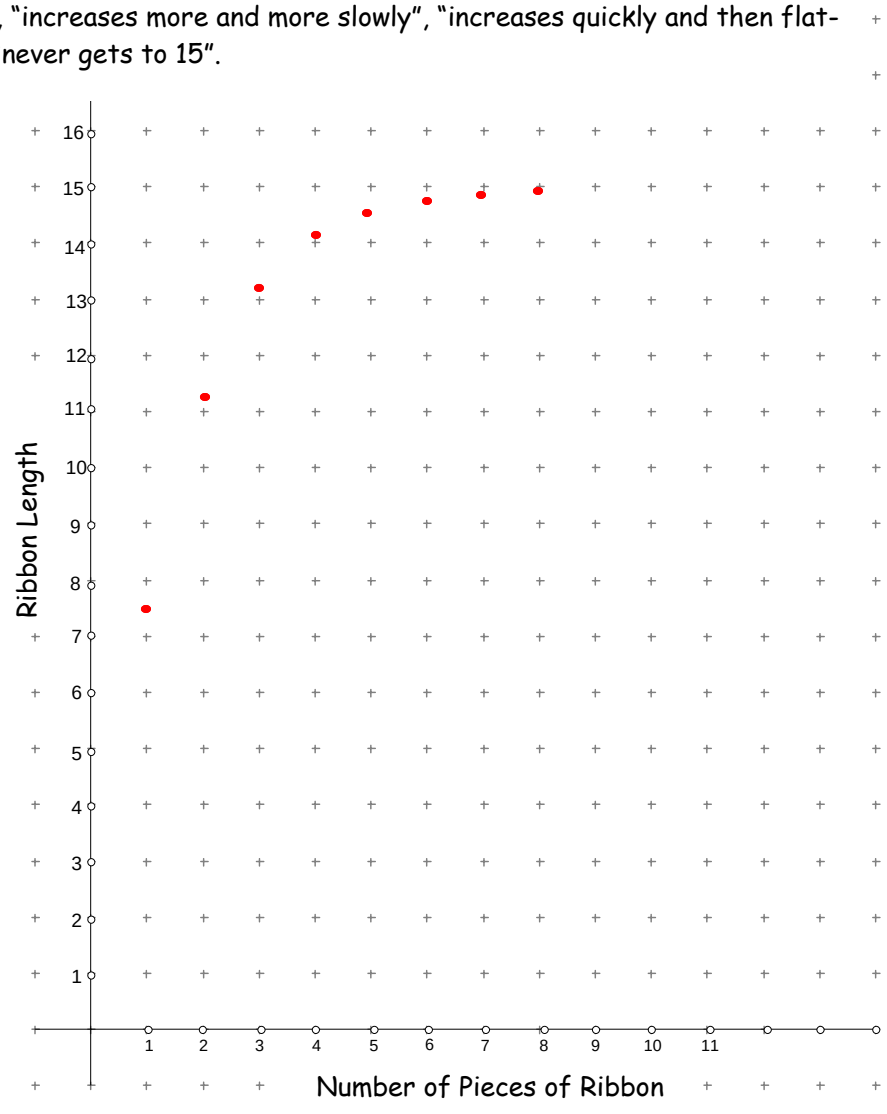
When describing this experiment, students should tell what they did and what happened to the ribbon as they cut it in half over and over again.

Materials Needed

1. 15—20 inches of ribbon for each pair of students. (A spool of 1/2 inch gift wrap ribbon works well.)
2. Scissors
3. Tape
4. A strip of construction paper or adding machine tape that is a little longer than the piece of ribbon
5. Measuring tape for each pair of students

When describing the graph in words, students may use phrases like, "curves downward", "increases more and more slowly", "increases quickly and then flattens out", "never gets to 15".

Number of pieces of	Length of ribbon
1	7.5
2	11.3
3	13.2
4	14
5	14.5
6	14.7
7	14.8
8	14.9



M2T2

Exploring Another Kind of Pattern

Participant Page

Journal Activity:
Describe the graph. Include answers to these questions in your description. Is it linear or nonlinear? Is it increasing or decreasing? Is the rate of increase or decrease constant? Is it symmetrical? Is there a maximum or a minimum?

Work with your partner to cut and measure the ribbon pieces following the instructions below. Record your measurements and graph the results. Analyze the graph. Is there a pattern? Describe the relationship.

_____

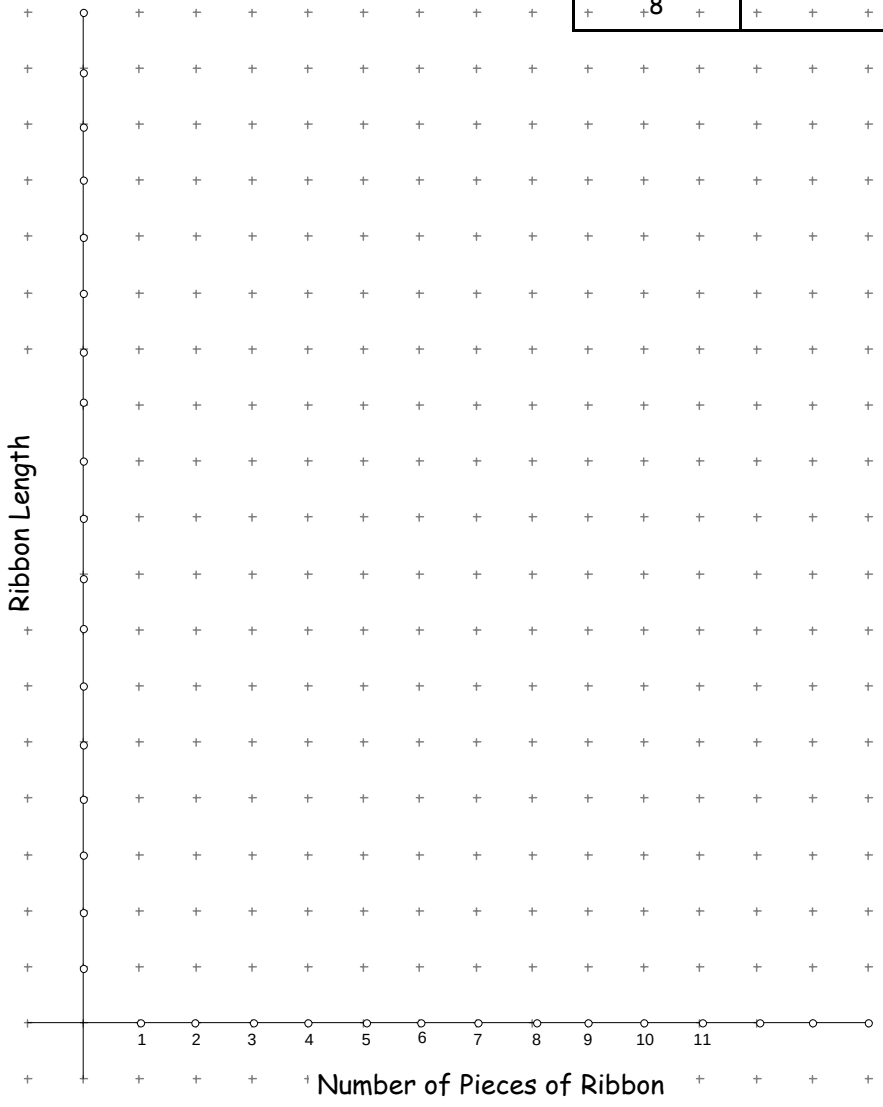
_____

_____

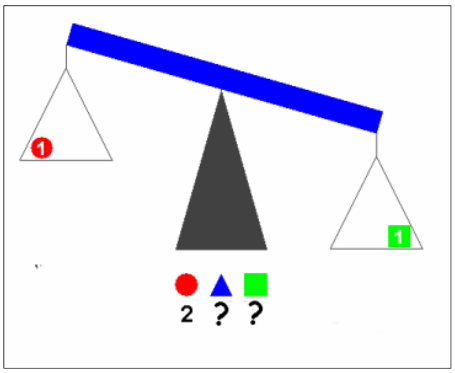
Number of pieces of ribbon	Length of ribbon
1	
2	
3	
4	
5	
6	
7	
8	

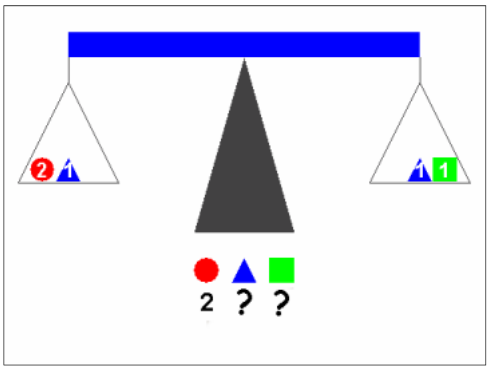
Fold your ribbon over on itself and cut it in half.

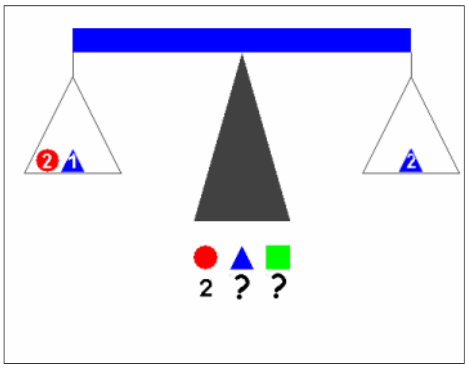
- Tape one of the pieces onto your construction paper strip. Measure it and record the length of this one piece of ribbon .
- Cut the other piece of ribbon in half. Carefully tape one of new pieces to the construction paper strip matching the end to the ribbon that is already there. Measure the total length and record.
- Cut the remaining piece of ribbon in half. Carefully tape one of new pieces to the construction paper strip matching the end to the ribbon that is already there. Measure the total length and record.
- Continue this procedure until you can't fold the piece that is left.
- Graph the ordered pairs.
- Describe the experiment and the graph in your journal.



Solutions



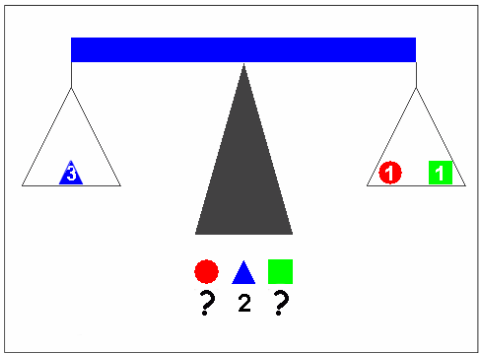


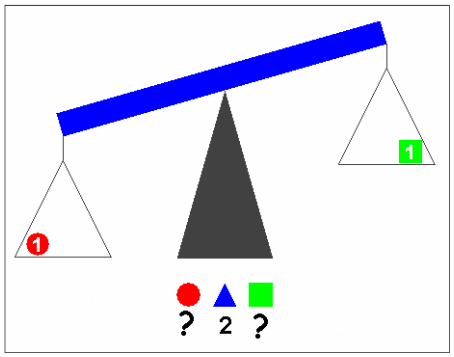


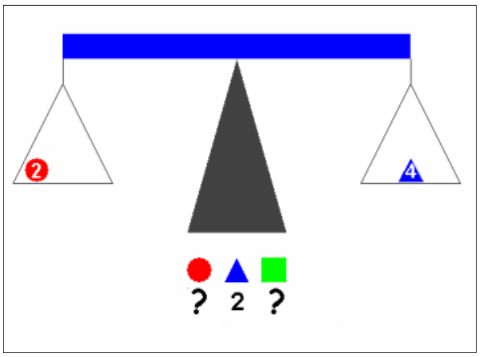
■ 4

▲ 4

● 2



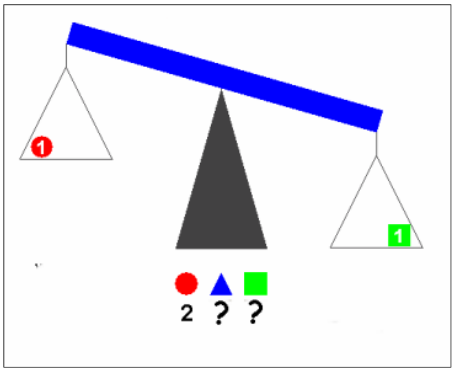


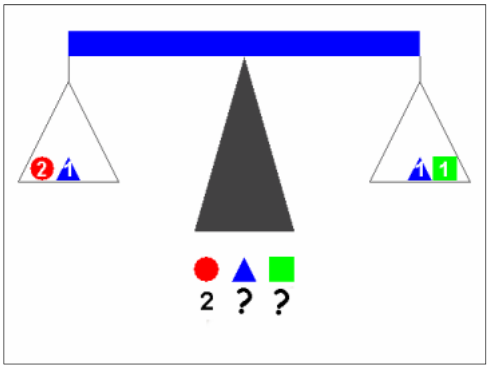


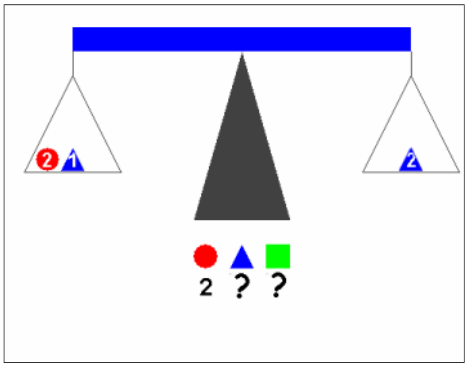
■ 2

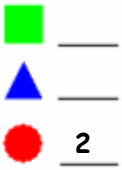
▲ 2

● 4



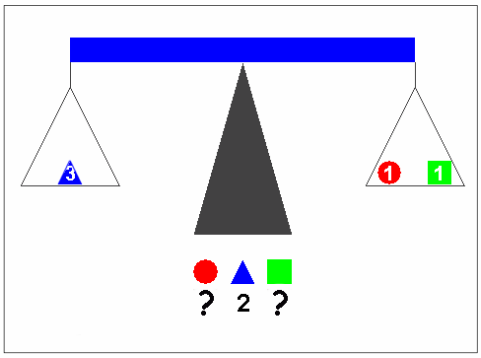


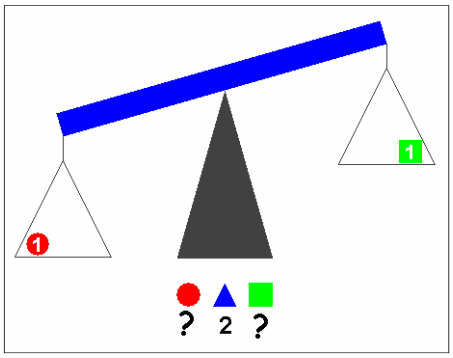


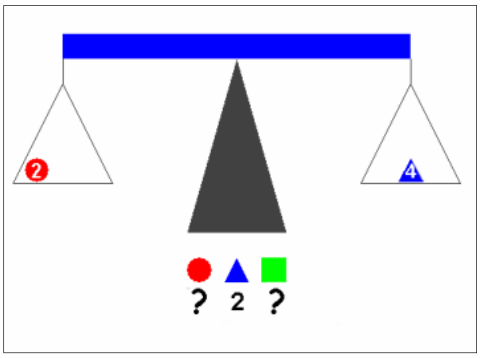


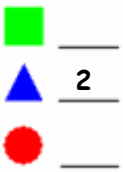
Determine the value of each objec from the information in the pictures. The values can only be whole numbers from 1 to 6. In the first picture, the red circle has a value of two units. One square is heavier than one circle.

Use the space below to explain how you got your answers:_____









Determine the value of the objects from the information in the pictures. The values can only be whole numbers from 1 to 6. In the first picture, the blue triangle has a value of two units.

Use the space below to explain how you got your answers:_____

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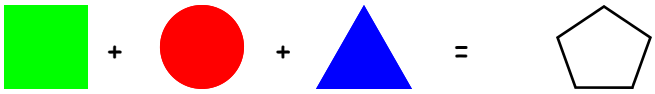
Participant Page

Shape Sentences Transparency

If the green square has a value of 2, the blue triangle has a value of 5, and the red circle has a value of 3, find the value of the pentagon.

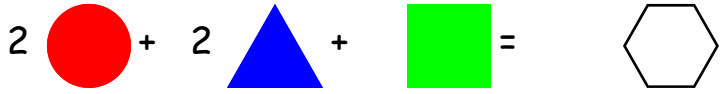


If the green square has a value of 5, the blue triangle has a value of 7, and the red circle has a value of 2, find the value of the hexagon.

Does the value of the hexagon change if we change the order of the colored shapes? _____

Another, shorter, way to show two red circles is to write a number in front of the shape.

Using this form we can show the hexagon like this:



If the green square has a value of 4, the blue triangle has a value of 9, and the red circle has a value of 2, make a shape sentence that has a value of 72. Write at least four different ways to do this.

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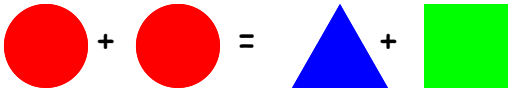
Participant Page

Mystery Shape Sentences Transparency

If the values of some of the shapes are given, find the value of the mystery shape.

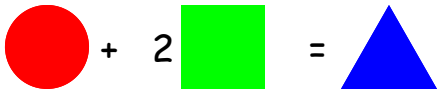
If the value of the red circle is 10, what is the value of the green square? _____

If the value of the red circle is 5 and the value blue triangle is 3, what is the value of the green square?
_____

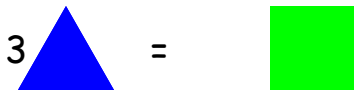


If the value of the red circle is 4 and the green square is 2, what is the value of the blue triangle ?
_____

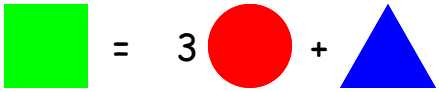
Find the values of the mystery shapes. If there is more than one possible value, give them all.



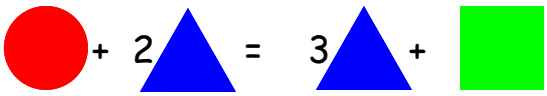
blue triangle = 7 red circle = 3 green square = _____



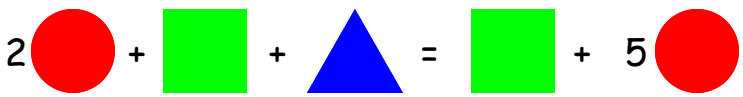
blue triangle = _____ green square = 12



blue triangle = 4 red circle = _____ green square = 10



blue triangle = _____ red circle = 5 green square = 2



blue triangle = 9
red circle = _____
green square = 3

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More Fun With Number Tricks

Show how these number tricks work with symbols and with algebra.

	NUMBERS	SYMBOLS	ALGEBRA
Choose a number			
Add three			
Multiply by two			
Add four			
Divide by two			
Subtract the number you first thought of			
The result is five.			

	NUMBERS	SYMBOLS	ALGEBRA
Choose a number			
Add the next larger number			
Add seven			
Divide by two			
Subtract the number first thought of			
The result is four			

More and More Fun With Number Tricks

Show how these number tricks work with symbols and with algebra.

	NUMBERS	SYMBOLS	ALGEBRA
Choose a number			
Double it			
Add nine			
Add the number first thought of			
Divide by three			
Add four			
Subtract the number first thought of			
The result is seven.			

	NUMBERS	SYMBOLS	ALGEBRA
Choose a number			
Triple it			
Add the number one larger than the number first thought of			
Add eleven			
Divide by four			
Subtract three			
The result is the original number			

Even More Fun With Number Tricks

Make up your own number trick. Prove it using symbols and with algebra.

	NUMBERS	SYMBOLS	ALGEBRA
Choose a number			
.			

Try your number trick using a decimal, fraction, or negative number. Does it still work?

	Decimal	Fraction	Negative Number
Choose a number			
.			

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Draw a picture of each stage of growth	Number of days old	Number of blocks
	1	
	2	
	3	
	4	
	5	
Students don't need to draw pictures beyond 5 days old		
	10	
	100	

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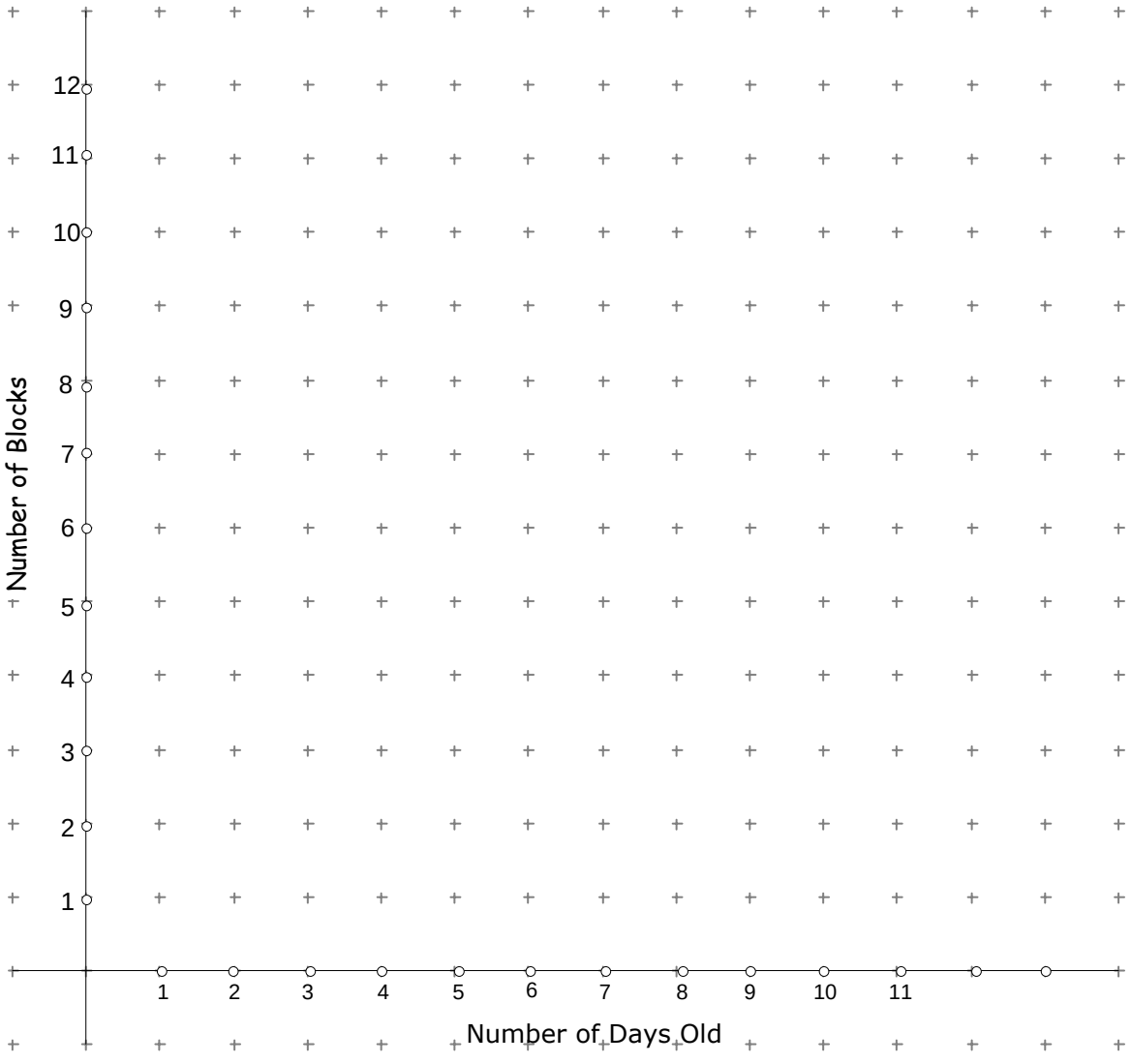
Number of days old	Number of blocks
1	
2	
3	
4	
5	
6	
10	
100	



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Graphing and Describing Patterns

Draw your pattern for the first four days. Complete the table. Graph the coordinate points from the table. Use words to describe a rule for finding the number of blocks for a pattern of any age. Write an algebraic equation for the relationship between the age of the pattern and the number of blocks.

One day old pattern

Two day old pattern

Three day old pattern

Four day old pattern

Number of days old	Number of blocks
1	
2	
3	
4	
5	
6	
10	
100	

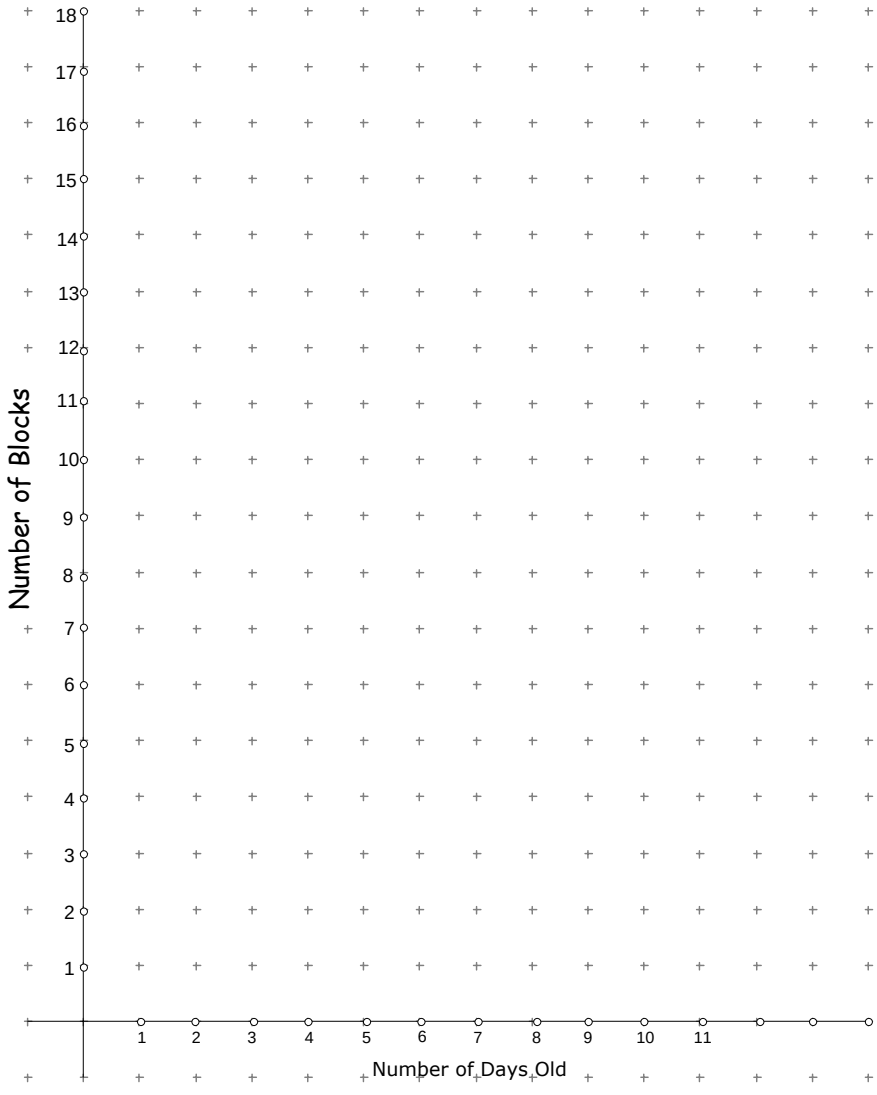
Describe the pattern. _____

_____

_____

_____

Write an algebraic equation describing the relationship between the age of the pattern and the number of blocks _____



Exploring A Pattern

Work with your partner to cut and measure the ribbon pieces following the instructions below. Record your measurements and graph the results. Analyze the graph. Is there a pattern? Describe the relationship with words and with an algebraic equation.

Fold your ribbon over on itself and cut it in half.

- Tape one of the pieces onto your construction paper strip. Measure it and record the length of this one piece of ribbon .
- Cut the other piece of ribbon in half. Carefully tape one of new pieces to the construction paper strip matching the end to the ribbon that is already there. Measure the total length and record.
- Cut the remaining piece of ribbon in half. Carefully tape one of new pieces to the construction paper strip matching the end to the ribbon that is already there. Measure the total length and record.
- Continue this procedure until you can't fold the piece that is left.
- Graph the ordered pairs.
- Describe the experiment and the graph here.

_____

_____

_____

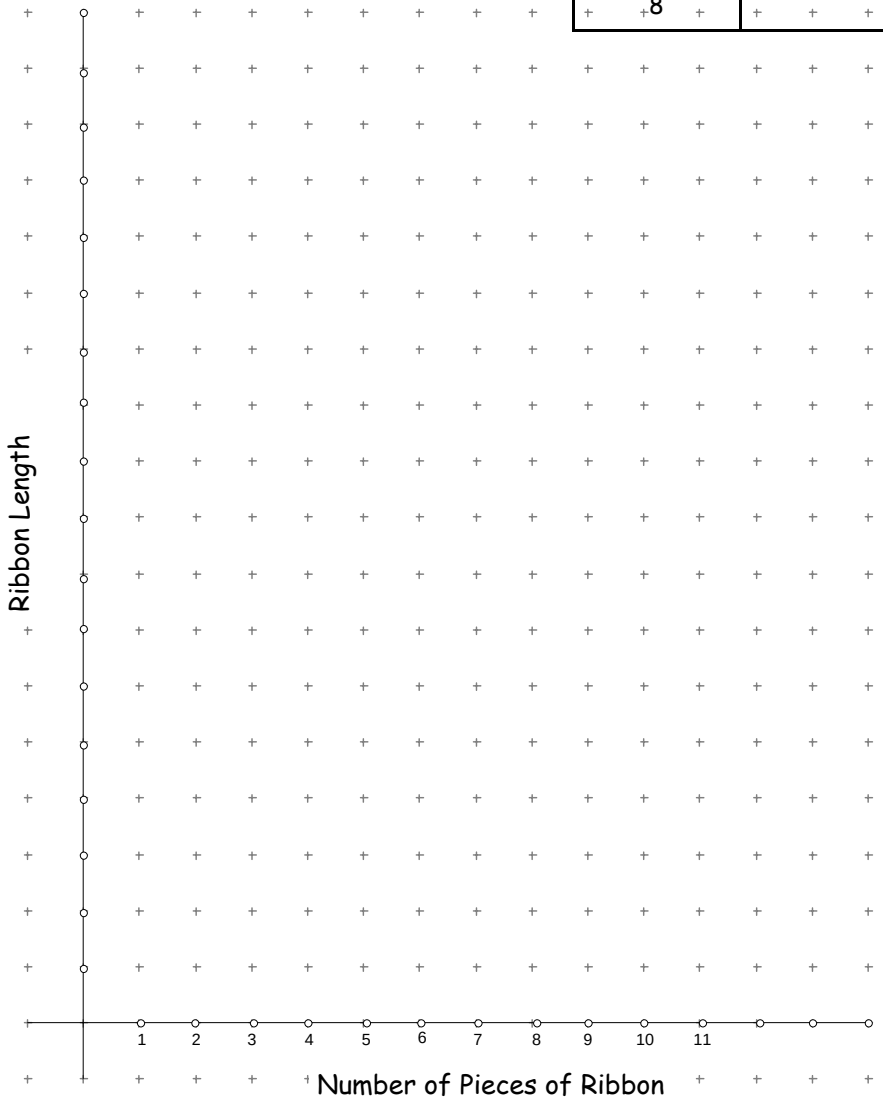
_____

_____

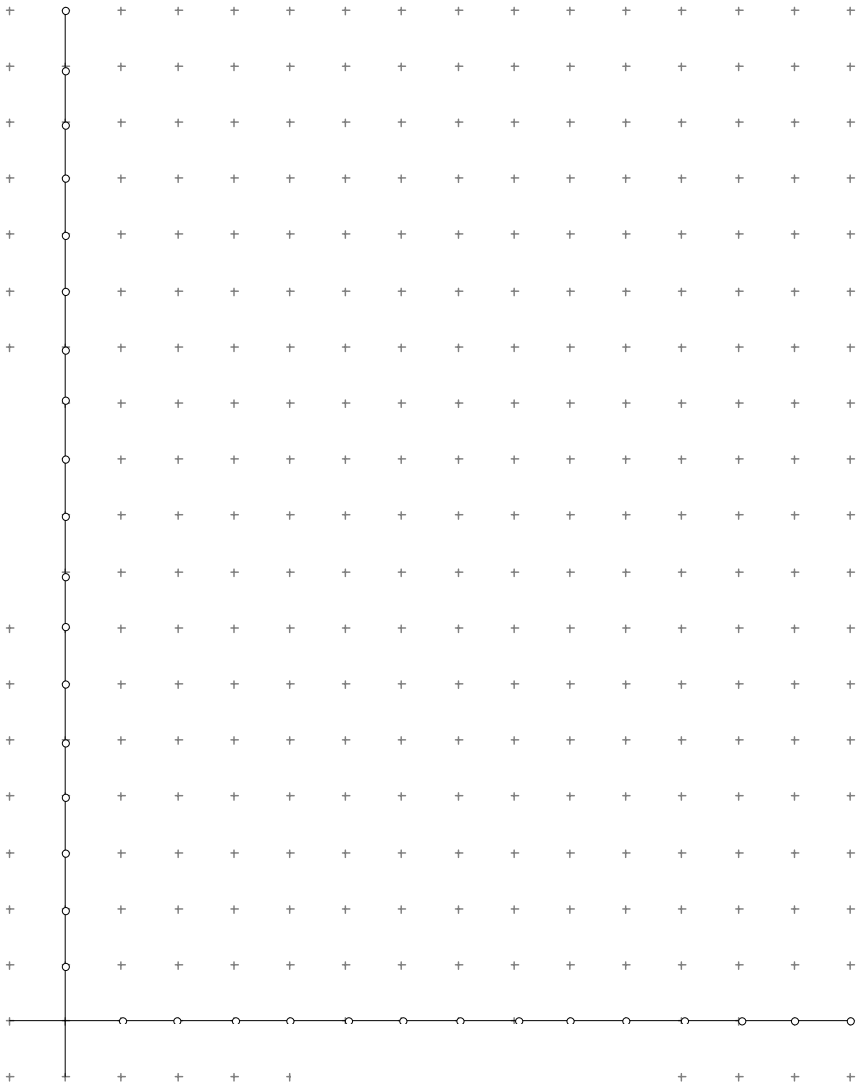
_____

_____

Number of pieces of ribbon	Length of ribbon
1	
2	
3	
4	
5	
6	
7	
8	



Exploring Linear Patterns



Email questions and comments to

Mozilla Firefox.lnk

Number Sense