To the memory of Leigh Burstein.
   Scholar. Visionary. Friend
In 1995, Dr. Luther Williams, the then Assistant Director for Education and Human Resources at the National Science Foundation, commissioned a panel to review the status of undergraduate mathematics, science, engineering and technical education and ‘provide a blueprint for the next ten years of National Science Foundation Leadership and programming at the undergraduate level’. The resulting document, ‘Shaping the Future: New Expectations for Undergraduate Education in Science, Mathematics, Engineering and Technology’ was published in 1996 (NSF report 96-139).

‘Shaping the Future’ (SF) concludes with recommendations for various audiences, including institutional and departmental administrators and faculty. Many of those recommendations provide a useful backdrop for the content of this present report—both in terms of the issues in undergraduate mathematics education that are presented here and the associated indicators (and data) that were developed in this feasibility study. Consequently, references to the Shaping the Future (SF) report are interspersed throughout this document. Following are illustrative recommendations:

- America’s undergraduates – all of them – must attain a higher level of competence in science, mathematics, engineering and technology. America’s institutions of higher learning must expect all students to learn more SMET, must no longer see study in those fields solely as narrow preparation for one specialized career, but must accept them as important to every student.

- America’s SMET faculty must actively engage those students preparing to become K-12 teachers; technicians; professional scientists, mathematicians, or engineers; business or public leaders; and other types of ‘knowledge workers’ and knowledgeable citizens. SF, page ii.

Note: ‘SMET’ refers to ‘Science, Mathematics, Engineering and Technology’.
Chapter Zero
Introduction and background

The postsecondary system is unique in its mission to create producers -- scientists, engineers, technologists and educators -- of science and technology who discover, synthesize and transmit knowledge that may enhance the quality of life and strengthen economic and social fabric. It also provides education and professional development for other citizens who use science and engineering in their everyday lives. (RED Indicators Report, 1995).

Undergraduate mathematics plays a pivotal role in our system of mathematics education. It is in college where our nation’s engineers and technicians are educated, where future scientists are recruited and where many of society’s leaders acquire basic quantitative skills. In addition, the instructional traditions of undergraduate mathematics form the model for all future teachers of mathematics. National Research Council. Moving beyond myths. P 13

0.1 Monitoring the quality of undergraduate mathematics education programs

Most mathematics department chairs, curriculum committees, and concerned faculty recognize that accomplishing their undergraduate mathematics education mission entails more than simply offering courses. The students to be served fall into a variety of categories that can be expected to include mathematics majors, majors in mathematics-intensive fields such as engineering, statistics, prospective K-12 teachers of mathematics, and those taking mathematics to satisfy general education requirements. Courses and programs must be planned to serve each type of student. Course content and sequence need to be carefully crafted to be suitable, effective, and efficient. For each course, qualified instructors (regular faculty, adjuncts, teaching assistants, etc.) must be assigned and appropriate, effective instructional approaches determined. Instructional factors to be taken into account include class size, use of discussion sections, selection of textbooks, and the role of technology such as graphing calculators and computer algebra software.

To be sure, there is a great variety of ways to carry out tasks such as those outlined above. Some courses work better than others — they have content that is appropriate to their target student audiences and are organized so the instruction can be effectively delivered. Some ways for identifying, developing, and assigning qualified instructors work better than others. Some instructional strategies may be more effective than others — or at least they are as effective as possible given limitations of available resources. In short, carrying out the responsibilities of a lower division mathematics education mission is complex and is typically accomplished with varying success. It is a demonstrable fact that departments with the same basic missions, and even offering similar course sequences, may vary significantly in the extent to which they succeed in accomplishing their respective undergraduate instructional missions.

At some point, those persons responsible for their department's instructional programs are likely to be led to ask the natural question, "How is our department doing?" Mathematics departments often proceed by inertia and tradition. At times they may be called on to evaluate and justify themselves formally to groups outside the department (for example, university required self-studies, program reviews, audits, etc.). At other times, certain department members may express the need to carry out a less formal assessment of 'how the department is doing', what needs are not being met, what courses are no longer effective, and so on.

In reality, evaluating the quality of a department's undergraduate teaching mission, and addressing those questions that must be answered in making such an evaluation, are often done on an ad hoc basis. However, this report is directed at those individuals who seek a more systematic approach to monitoring of
the quality of the undergraduate program and are looking for ideas as to how this responsibility can be carried out successfully. We therefore consider the question: "How can the quality of a department's undergraduate program be consistently, systematically, and effectively monitored to identify trends, problems, successes, and needed changes?"

The effective monitoring of program quality at the institutional level has a counterpart at the national level. Entities that help shape national priorities and policies in collegiate mathematics education (for example, funding agencies such as the National Science Foundation (NSF), or professional organizations such as the Conference Board of the Mathematical Sciences (CBMS), the American Mathematics Association for Two Year Colleges (AMATYC), the Mathematical Association of America (MAA), or the National Council of Teachers of Mathematics (NCTM)) need answers to many of the same questions as do persons at the institutional level. The major difference is that, in this case, the questions are asked about the aggregate of U.S. colleges and departments. It is the effectiveness of undergraduate mathematics instruction at the national level that is to be assessed. Methods are needed to identify and understand weaknesses, strengths, and needed changes for the aggregate, and thus in turn for the colleges that make up that aggregate. The question of how to consistently and systematically monitor programs at the national level, we argue, closely parallels the corresponding question at the individual institutional and departmental levels.

It is a major goal of the Indicators Project to assist mathematics departments as they gather, analyze and report data that will inform key indicators that may be used to assess various facets of their instructional programs.

0.2 What are indicators?

We are all familiar with the use of economic indicators to describe the health and direction of a nation's economy. These indicators — for example, the rate of inflation, the Dow-Jones Industrial Average, the gross domestic product, and others — reflect "performance characteristics" of the economy. Even when these data have complex relations with other aspects of the economy, they provide "benchmarks" — comparisons of the state of the economy with itself at different times. The meaning of those comparisons is often the subject of public discussion and of econometric models that are devised to explain how the benchmarks relate to the state of the economy. The significance of some of the data (for example, number of starts in new housing) is relatively uncomplicated. Sets of these indicators and associated benchmarks help inform judgments of the economy's strength and of the direction of its movement (that is, prediction of more likely economic trends by comparison with past performance of the economy).

Education indicators can serve similar purposes. Mathematics departments face questions of evaluating the status of their instructional programs (strengths and weaknesses) and identifying needed or suggested areas of possible improvement. Indicators can provide empirical data to assist mathematicians make informed judgments about the quality and direction of their instructional programs.

The education research literature has considerable information about statistical indicators and their use in providing empirical benchmarks that help in judging the quality of educational programs. The details of that literature are not appropriate for this report. However, some readers may find some of the ideas and techniques useful and wish to pursue it on their own. Part of our project (on which we report here) is to sift this educational literature, translate it into reasonable English with minimal jargon, and make a start at applying it to the monitoring and improvement of lower division mathematics instruction.

That literature contains varied discussions of what qualifies as an educational indicator. Shavelson, et al., have stated:

*Education indicators are single or composite statistics that reflect important aspects of the education system (as economic indicators reflect aspects of the economy). They are expected to tell a great deal about the entire system by reporting the condition of particularly significant features of it.... (An education indicator) should provide insight into the 'health', quality or
effectiveness of the system; and it should be useful in the educational policy context. (Shavelson, R., et al., 1987, page 8)

Most of the above statement is straightforward. However, some mathematicians may be surprised to hear that they work in an "educational policy context." But, shedding the jargon, anyone who has responsibility for seeing that undergraduate mathematics is taught, and taught well, makes decisions about how to get the job done with the resources available. As "everyday" as that seems, such decisions reflect stated, or implied, educational policies and those making the decisions would qualify as working in an educational policy context as defined in the above quotation.

Education indicators in the sense discussed so far serve at least three purposes:
- Indicators can help us interpret or understand what is done at the different levels that affect mathematics instruction. These levels include college-wide activity, department activities, specific educational programs (e.g., the calculus sequence) and individual mathematics classrooms (e.g., instructors' classroom uses of technology or students' grasp of the basic ideas of the calculus). Indicators can be used in a way that facilitates comparing, reliably and objectively, what happens in different classrooms and courses so we can more easily determine what works well and what works less well.
- Indicators can help us monitor trends, that is, changes over time in what happens in mathematics classes, courses, and departments (for example, enrollment patterns of females in mathematics-based career tracks).
- Indicators can help us determine the effects of deliberate changes we make in instruction, courses, and departmental or college-wide policies (for example, moving to larger class sizes in calculus so that all sections can be taught by faculty rather than teaching assistants or adjuncts, adopting a particular reformed calculus approach, instituting a college-wide mathematics general education requirement, and so on).

Major goals of mathematics departments 'self-evaluations' often include: (1) helping us understand what is going on in carrying out service and other instruction for which the department is responsible and (2) evaluating those gradual, unplanned changes as well as deliberate, planned changes (changes in policy as well as changes in practice). Indicators can be of help in all these tasks.

During the past few years, indicators have received increasing attention at the institutional level, in their role as a source of data for accountability purposes. For example, in South Carolina, a law was passed by the State Legislature in 1996 to the effect that state appropriations for the public colleges were to be based entirely on how well each institution was judged to perform. Some nine different areas and more than 30 indicators were developed for this purpose, including the following:\footnote{This quotation uses the following format: area (indicators). So, for example, in the area of quality of faculty these indicators were developed: credentials; faculty reviews that include student and peer evaluation.}

Mission (an acceptable mission statement and a strategic plan based on it; Quality of faculty (credentials; faculty reviews that include student and peer evaluation); Instructional quality (institutional emphasis on teaching quality and reform); Institutional cooperation and collaboration (sharing equipment and other resources within and between institutions, and with the private sector); Administrative efficiency (Amount spent on administration vs. academic programs); Entrance requirements (Credentials of students, including standardized-test scores and high school standing); Graduates' achievements (Graduation rates; employer feedback); User-friendliness (accessibility to the state's citizens); Research spending (spending on reforms in teacher education). (Chronicle of Higher Education, April 4, 1997, p. A26)
0.3 The utility of indicators in monitoring program quality

The questions faced by mathematics department chairs and faculty seeking information to describe and evaluate the current status of their programs require factual, data-based answers. Arguably, many aspects of carrying out a department's teaching mission, just as much of teaching a course effectively, remain a matter of art and experience. For departments and for individual courses, experienced instructors' opinions do indeed matter. However, even this accumulated wisdom can lead to more insightful conclusions when informed by appropriate data describing what actually occurs in the life of the department.

The kinds of questions that are faced likely have to do with at least some of the following: the department itself—its goals and priorities; the curriculum (programs and courses); the instructional staff; the classroom practices commonly found in the department; and the students served by the department. These aspects of undergraduate instruction become focal points for data that are needed to inform experienced opinion and to help a department to accurately identify its teaching strengths, weaknesses, and needs.

Much of these data may consist of detailed, narrative (qualitative) information about specific courses, instructional practices, student attitudes, and so on. We contend, however, that these data should also include numerical (quantitative) information about various aspects of a department's programs. Statistical measures that are used to inform evaluations in this way are called indicators. The current report gives the results of a project to explore the feasibility of developing a set of indicators for monitoring the quality of undergraduate mathematics programs.

The results of this project should be useful to persons who through careful, systematic evaluation desire to improve the quality of undergraduate instruction in their departments. The outcomes of this project, generalized and abstracted from the concrete details of departmental life, may also assist those at the national level who are concerned with priorities and policies to be followed in improving the quality of undergraduate mathematics education. In summary, therefore, descriptive statistical indicators that help inform departments as they evaluate themselves, may also serve to help profile and monitor the national status and needs of undergraduate mathematics education.

It is important to note that indicators are used to help present a picture — in this case, of what takes place in the life of a department as mathematics is taught. Hence, we seek not single, isolated statistical measures, but carefully organized sets of indicators. To help us identify what should make up these sets of data, we propose a framework or model within which such indicators can be developed and organized. Finally, we provide selected examples of illustrative indicators that might be used in evaluating the educational quality of the first two years of undergraduate mathematics.

The exemplars that we provide are obviously far from a complete set for even simple purposes of monitoring program quality. They are intended to serve only as catalysts for developing more complete networks of data.

0.4 Factors and questions guiding indicator development

As we evaluate our mathematics programs using a combination of professional experience and judgment as well as empirical (indicator) data, we are likely to focus repeatedly on those factors believed to affect undergraduate instruction. Within each focus, we ask the same types of questions. If we can identify these focuses and factors, as well as essential questions about each, we are well on our way to developing a useful set of educational indicators.

Indicators are of necessity selective. They picture only certain features of what we do in providing mathematics instruction. If they weren't selective, we would drown in a sea of empirical information.
Because indicators are selective, we must be sure to select the important features of what we are doing so that the data we obtain will be relevant and helpful in our decision-making. That's why identifying foci and questions before planning what data we will collect is such an important first step.

In this chapter, we identify five factors on which to focus when thinking about the effectiveness of undergraduate instruction. These factors are: (1) the institution (two year college, comprehensive university, research university, etc.) and the department; (2) the mathematics curriculum; (3) the instructors (including faculty, teaching assistants, adjuncts, etc.); (4) the classroom (that is, the instruction and the assessment of what students are learning that goes on in individual classes); and (5) the students in the classes and activities through which undergraduate teaching is provided.

For each of these factors there are key questions to be addressed as we monitor the quality of mathematics teaching and learning. We consider each in turn.

The Institution and Department. The success of a mathematics program is determined in part by what it is trying to accomplish. To monitor and understand this, we need answers to such questions as:

- What are the major goals of our department (not just in teaching and not just with undergraduates)?
- What role does our undergraduate program play in these goals and what priority does it have among the many things we hope to accomplish (research, representing the mathematics community, graduate instruction, etc.)?
- To what extent do the goals and priorities of the department align with those of the institution?

These are certainly not the only questions we would want answered in order to understand those aspects of our college and of our mathematics department that affect undergraduate instruction. However, they are two important questions that can serve as examples here. We would also want to consider other factors such as available resources, what regular methods (if any) are available to help us decide when courses or instructional approaches are needed or are no longer effective, the commitment of the instructional staff to the continued improvement of the undergraduate program and the demonstrated intention of the department to support the continuing professional development of its instructional staff as a departmental and institutional asset (See Ewell, 1994).

The Curriculum. The traditional meaning of 'curriculum' is the course of study provided. College mathematics instruction usually is packaged into courses and sequences of courses that deal with particular mathematical content and have specific goals and, especially for lower division courses, 'official' instructional approaches to accomplish each course's goals. Mathematics departments (and, in some cases, other divisions of a college or university) are responsible for dealing with those courses, their mathematical content, goals, and approaches. Let's call that combination of courses, sequences, course goals, and expected instructional approaches the 'curriculum'. Given this definition, questions such as the following arise:

- How does our curriculum relate to the goals of our department, to the requirements of our 'partner disciplines' (such as engineering or science) and to the needs of our students?
- What methods (if any) do we use to monitor how well our curriculum is accomplishing its goals? Who is responsible for this monitoring (department chairs, curriculum committees; course coordinators, individual instructors, etc.)?

Certainly there are other things about an institution's mathematics curriculum that we would want to know. But these two questions are illustrative of the kinds of information one would seek as quality concerns about the curriculum are explored.

The Faculty (instructional workforce). Departments and institutions have goals or missions. The instructional staff refines and makes explicit those goals through actual classroom instruction. Obviously, a
department's instructional staff (its qualifications, experience in teaching, beliefs, and many other factors) greatly affects the quality of the department's programs. Two sample questions are:

- **How do our faculty's interests relate to major components of our undergraduate programs?** Do the fulltime, tenure-track faculty share an interest in and responsibility for lower division instruction? Do they willingly and routinely help provide this instruction?
- **What professional development activities for the faculty have taken place in the past three years?**

'Professional development', denotes that faculty are professionals and, more specifically, points to the fact that they are professional teachers as well as mathematicians. The demands, possibilities, and approaches for effective undergraduate mathematics instruction gradually evolve and grow. The best of current teaching practices can continue to be enhanced by taking into account, for example, recent developments in knowledge about how mathematics is learned or by becoming familiar with research on instructional strategies (such as the efficacy of using graphing calculators as a problem-solving tool).

Teaching professionals do their best work when they are informed and aware of changes and new possibilities. 'Professional development' is shorthand for activities that are made available to the instructional staff to help them stay informed and equipped to consider the best approaches, old and new, that might be useful in carrying out their teaching responsibilities.

**The Classroom.** 'Classroom' here is shorthand for what actually happens between instructors and students as teaching is carried out and learning takes place. Obviously this category includes many kinds of questions for which actual data are helpful in providing answers.

Classrooms are the "center stage" of undergraduate mathematics instruction. Many activities take place there in fulfilling teaching missions. In particular, instructional activities are carried out and student learning is assessed through tests, homework, the instructor's judgment, and so on.

Two sample subclasses of questions are considered here.

**Classroom instruction.** To be sure, actual mathematics instruction is the central activity of carrying out a department's teaching mission. A department's plans, uses of resources, policies, and self-evaluation are all aimed at getting qualified instructors into classes appropriate to the students served and in which the instructors use effective methods to provide activities that help students willing to take advantage of these opportunities to master the goals of that class. In considering the kinds of activities that create opportunities as a part of instruction, several questions are relevant. Here are two:

- **To what extent are students active participants in the ongoing classroom work rather than passive observers?**
- **What kinds of technology are available to support and enhance classroom instruction?** To what extent is the available technology used and in what ways is it used? Does the department have policies about its use? Does the department help the instructional staff make more effective use of the available technology?

To be sure, these two questions are quite different from each other. Yet they are related in that creative uses of technology can be helpful in promoting student engagement in the subject matter at hand (See for example, Kaput, 1992).

**Assessing student learning.** In addition to teaching, instructors regularly assess, both formally and informally, how well their students are learning. Instructors do so for formal purposes of marking and assigning grades to students. They do so less formally in many cases to see if their teaching is effective, if the message is getting through, if changes are needed, if the instructional pace needs to be picked up or slowed down, and so forth. Even the most traditional, lecture-oriented instructors gauge their "audience" and often make at least small changes in response to what they see. This idea of assessing in the classroom what students are learning leads to an entirely different range of classroom-related questions. Two examples are:

- **What kinds, if any, of formal (collected and marked) assessment activities are in use that go beyond the usual kinds of tests?** 'The usual' include quizzes, chapter
tests, and department mid-term and final examinations. What other kinds of assessment activities are used in our department? Are projects assigned? What written work is required, apart from numerical or symbolic answers to exercises?

- **Is assessment used to promote learning rather than to simply assign grades to students and, if so, how?** Various assessment methods are available for deciding what, how much, and how well students know the mathematics that was taught. But is assessment used only for formal purposes to assign grades? Are our assessment methods also used to provide the instructor with a picture of what students do and don't understand so that the instructor can tailor her or his future activities to where the students are mathematically? (See, for example, Schoenfeld and Dossey, 1996.)

Clearly these latter questions make it clear that gathering information and judging what and how well students are learning can be quite complex. Most of these complexities can be ignored or they can be addressed in an attempt to improve instruction. The specifics of whether and how these complexities are addressed may tell a department important things about its effectiveness, especially when their department's descriptive data are compared and contrasted with the benchmarks of corresponding data from other institutions known to be effective. It is this kind of comparative benchmarking that suggests that national and aggregate collecting of indicator data may help local departments as well as those who help to shape national policy for college mathematics.

**The Students.** Students are the target audience that must be reached in order for a department to successfully carry out its educational mission. Certainly students who do not do well in mathematics bear some, if not most, of the responsibility for their lack of success. But instructors, departments, and institutions must each take some share in that responsibility. Given this assumption, questions about how students engage in and react to mathematics instruction are another important focus in evaluating the quality of a department's instructional program. Two sample questions that lead to empirical data serving as indicators include:

- **What opportunities are provided for our undergraduate students to take part in the scholarly and social life of the department?** For many departments, teaching is service and students are a part of "classroom life" only. In other cases, departments seek to involve their majors, beginning students, and others in non-classroom aspects of the department's life, either by encouraging participation in mathematical, scholarly activities, by providing social opportunities in which students are encouraged to participate, or both. To help evaluate our program's effectiveness and how it is accomplished, we need to ask, "Are opportunities other than those in classrooms available to our students? Are the students encouraged to share in those activities? To what extent are all students rather than only selected subsets, such as mathematics majors provided such opportunities?"

- **How do our students, present and past, feel about our department and its programs?** Do they feel well served by our courses? Are they getting or did they get what they needed to master the mathematics necessary for their chosen career? Do they feel challenged? Do they feel supported? Do they consider the department and its faculty easy to approach? Do they consider the program a necessary evil or an important opportunity? How do they feel about specific courses and programs? Do our graduated majors have any suggestions for how we might have provided them with better preparation?

As before, these questions and the areas they represent are meant only as samples of the kinds of issues a department might wish to consider in evaluating the quality of what it is doing for undergraduates. No one department is likely to seek answers to all these questions but each department likely has a somewhat similar set of questions to be answered.
Questions such as those in the five focus areas above are often, if not typically, answered in the absence of reliable empirical data. This is not intended to belittle the professional judgment of mathematicians. However, even the most expert judges arrive at better conclusions when informed by accurate data. Decisions about program quality, student outcomes, and staffing too often are based on anecdotal information, tradition, or conjecture.

We as mathematicians are not exempt from at times approaching these kinds of questions at times without seriously engaging our professional experience and expertise. There are many reasons for this, not the least of which is that we have so many other pressing demands on our time. However, accurate and informed answers to these and similar questions are essential as a basis for building and maintaining high quality undergraduate instruction programs. A carefully designed and well developed indicator set and its consistent use can go far toward providing needed direction in developing, monitoring and improving our instructional programs.

0.5 The importance of indicator sets rather than single indicators

The categories and questions above suggest how wide is the range of potential indicators to aid in the careful collection of data to enable mathematics departments to make more informed decisions. The range of factors and their associated statistical indicators to be considered are bewilderingly broad. If indicators are to provide useful guidance, selections must be made and organized to provide a set that portrays an accurate, purposeful and integrated picture.

As already noted, it is important from the very beginning of their development, that indicators be viewed as occurring in sets rather than as isolated bits of data. Indicator-based portrayals of mathematics instruction are inherently composites.

Developing a set of indicators for undergraduate mathematics requires careful thought and planning. The operative word here is 'set'. A single indicator for a complex enterprise such as carrying out a teaching mission would likely be misleading and subject to erroneous interpretations. What is much more useful to departments is, instead, a web of related indicators enabling the targeting of areas of importance.

Let us consider one example of the danger of focusing on a single indicator rather than a planned, organized set of related indicators. Figure 0.1 shows how many students are retained (or, dually, how many drop out) across educational levels of study from grade school to graduate school. The simple story appears to be the massive drop off in mathematics study (notice that the vertical scale is logarithmic). The jargon for how many students are retained over time is 'retentivity'.

Figure 0.1 appears at first to tell a simple story. Further consideration suggests that the obvious interpretation may be misleadingly simple. What appears as a lack of retentivity may be, in fact, a result of increasing selectivity. The criteria for being allowed to continue mathematics study may become increasingly demanding as we move from pre-college to college and from undergraduate to the two levels of post-graduate study. This single indicator may portray increasing selectivity but it cannot alone reveal other aspects of such selectivity. Does increasing selectivity also involve other changes? For example, does it change the composition of the students studying mathematics at later points? Does it reflect different proportions by gender or by ethnic background? That is, are some groups more "at risk" than others as the criteria for continued mathematics study become more demanding (both in the sense of official policies and in the form of demands for more resources to allow one to continue mathematics study)? Are certain courses (for example, the calculus sequence) major "gatekeepers" for the continued study of mathematics and thus more in need of informed change than other courses, or should changes in selectivity be across the board for all courses, perhaps by admission standards at a college-wide level?

If we modify our policies to retain more students and decrease this selectivity, what other consequences will derive from this policy change? Will simply "opening up the pipeline" be enough to increase participation by traditionally underrepresented groups? Will such an "opening up" result in weaker standards and therefore a less adequate mathematical preparation for all? To what extent can we retain larger numbers of students, especially from underrepresented groups, and still maintain our standards for expected mathematics mastery? Could greater retention be accomplished by focusing on key, "gatekeeper" courses and sequences rather than by across the board changes in selectivity?

If an institution looks only at its drop-out rate (a single indicator), without regard for the type of students or the differing impact of various courses, the picture that emerges probably hides as much or more than it reveals and changes based on these data would be misguided. If that institution assumes a simple relationship between the student dropout rate and a corresponding, undifferentiated index for
selectivity of admissions, then neither the data nor what they seem to reveal about a key relationship will likely inform an effective policy.

Global indicators such as retention rates and selectivity of admissions can provide important information about the 'health' of a program, department, or institution. However informed decision-making and planning requires knowing more about other factors that are associated with enrollments and successful completion of particular courses by various types of students. A set of more specific, related indicators on drop-out rates and on the effects of selective admissions allows us to more precisely identify and understand problem areas that call for special attention. Indicators need to be organized into related, integrated sets in order to inform not mis-inform our plans for effective mathematics programs.

It is also important that indicators be interpreted in context. The setting from which indicator data are obtained may be important to understanding the significance of the indicator and for selecting appropriate benchmarks. For example, comparing budget allocations for undergraduate mathematics in a small four-year liberal arts college to related allocations in a large research university would necessarily be misleading, regardless of whether it seemed to favor higher or lower budgets. The two types of institutions have different overall missions and, in particular, different responsibilities for providing mathematics instruction.

Various mathematics teaching missions have differing budgetary implications. Were we to change the proportion of an institution's budget allocated to mathematics instruction or even the proportion of the institution's mathematics department's budget devoted to undergraduate instruction, the results would remain fundamentally incomparable because of the differing priorities for instructing undergraduates in the two types of colleges. However, for comparing institutions with similar missions, the same budgetary allocation indicator could provide useful benchmark data and have considerable interpretive power.

Single indicators are best used to raise questions or to identify potential problems or issues. They are less useful for rendering overall judgments about the how adequate a program or institution is or how well it performs. That latter purpose is best served by sets of related indicators that provide more specific, detailed information and that serve as context for each other in informing the common sense of experienced professionals. Even sets of indicators are best used as sources of information for reflection rather than decision criteria. Used in this thoughtful way, they can lead to more insightful planning and decisions.

Given these considerations we must conclude that indicators need to be carefully constructed in structured sets of related indicators that provide full, rich, and contextually sufficient pictures of programs. This is true whether the indicators are used for external or internal analysis and evaluation. Externally, demands for indicator data traditionally have focused on key instructional outcomes and insight into how instruction is organized and delivered. This concern has been accompanied by calls for evaluating the "value added" by mathematics instructional programs. Unfortunately, requests for data on how much "value" is added by a department or institution's efforts have often used a simplistic "input-output" model with little attention paid to the qualitative differences by which programs add value to their "inputs" — that is, to student demographics, availability of instructional resources, whether the instructional staff keeps current in terms of classroom uses of technology, and so on. Accurately picturing what our departments accomplish (their "outputs") through their teaching efforts, given what they have to work with (their "inputs"), must take into account not only "how much" (numbers of students entering or completing calculus, continuing in mathematics-intensive majors, etc.) but "how well" (what students can actually do mathematically after instruction that they would not do before).

Growing fiscal pressures and changes in student enrollments and what those students seek from mathematics programs are leading to increased internal evaluation of undergraduate teaching as many colleges and universities take new looks at the mathematics instruction that they deliver. Evaluation in that demanding context obviously will not have much effect if based solely on anecdote and opinion. Empirical data are essential for making convincing cases for the worth of what we do. These evaluations also seem to have moved from focusing only on "inspecting the end results" of mathematics instruction to considering more broadly how this is accomplished and how efficiently and effectively the means are used. This broadened focus makes data from well-conceived, structured indicator sets even more important.
0.6 A Model for Undergraduate Mathematics Indicators

How can we develop the kinds of organized, integrated indicator sets we have been arguing for? One effective method is to use a generic planning model that systematically identifies the factors to be considered and their relations to each other. Figure 0.2 presents a schematic overview of a model that provides a framework for thinking about related indicators that help to describe the goals, status, and quality of undergraduate mathematics programs. Using this framework not only suggests areas needing indicator data but also helps to identify areas missing from our consideration.

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Figure 0.2: An Organizational Framework for developing indicators for mathematics program quality

In the model, five clusters of issues related to undergraduate mathematics are identified. These major components are indicated in the left-most column of Figure 0.2: department, curriculum, faculty (instructional staff), classroom practices, and students. These are, in fact, the focuses we identified in the earlier discussion.

The column's topic order suggests a 'top down' view of undergraduate mathematics education, starting with a consideration of the department in its institutional setting. Mathematics education takes place in a variety of post-secondary institutions. Within these institutions, with different missions, departments have differing goals and priorities. It is the department that organizes programs, develops mathematics curricula, selects instructors, staffs courses, provides educational opportunities for students, assesses student progress, conducts research, and otherwise provides much of the context within which the teaching and learning of mathematics take place. That is, mathematics departments make plans and create environments for the other levels of mathematics instruction, noted in the remaining four rows in Figure 0.2.

Now let us refer to the columns of Figure 0.2, labeled 'Initial conditions', 'Actions' and 'Outcomes'. For each of the five levels (rows of Figure 0.2), there are three aspects of that level that may be considered. These are specified by the three columns of Figure 0.2. Certain conditions occur before the actions taken. These we label 'Initial conditions'. Actions have results that we label "Outcomes". Between the stage set by the initial conditions and the outcomes in that setting are the actions themselves, a sort of transaction among participants whether those participants are the persons in seeking to change a curriculum or course, the instructors and students interacting in a classroom, or whatever players are appropriate for the actions taken in that setting. These factors we label "actions".

Figure 0.2's diagram has three columns that reflect these aspects that shape a mathematics program's activities — from institutional and department intentions to evaluating student outcomes. The
first column focuses on the initial conditions and contexts for each of the five levels of undergraduate mathematics programs. For example, at the department level (Level I), a department's goals are conditions flowing from the teaching mission and priorities assigned it by the institution of which it is a part. Column Two focuses on the actions by which initial conditions moves toward program outcomes. An example, again at Level I, might be having a departmental committee that regularly reviews that department's goals and priorities. Column Three represents the outcomes of actions that take place in the setting defined by the initial conditions. Continuing our example, at Level I this might be statements of a department's priorities that emerge from reviewing the department's missions and goals.

The fifteen cells of Figure 0.2 are created by combining the five levels and the three evaluative viewpoints that we have just described. Cell (1,1), for example, specifies indicators that reflect a program's current goals or intentions, and the charge given to it by the broader institution. This cell might also include indicators on the recency, breadth, and consistency of those goals and intentions. Cell (1,2) includes departmental actions to plan what to do to carry out their assigned mission and meet those goals. This could include the departmental structure for monitoring whether goals are attained and for initiating new plans to better attain or revise goals based on past attempts to accomplish them. Cell (1,3) would encompass indicators whose focus is on whether departmental goals are attained. For example, it might include data on to what degree the program's various goals are being met and by whom.

Similar descriptions hold for each of the other four levels (the rows, Levels II through V) of an undergraduate program — that is, for curriculum, faculty (instructional staff), classroom practices, and students. We conclude this chapter with exemplars of indicators categorized by a few selected cells of the model in Figure 0.2. These indicators might well be included in an initial set for departments to consider for use. More importantly, they should help make the idea of an indicator more concrete and be of assistance as mathematics departments undertake the design of an indicator system to study and evaluate their own structure and functioning.

0.7 Issues and Associated Indicators

In the project, 10 issues (plus Issue 0, Demographics) as listed below were identified. For each issue, approximately six indicators were developed.

**Issue 0 -- Department Demographics**
(For details, refer to Chapter One below)

To begin to understand a department, one must have a feel for its composition. Thus, one of the first kinds of data that should be considered are those that give a demographic profile of the department. For example, it is important to know the make up of the *instructional staff*: Its overall size as well as its composition by rank, gender, ethnicity, and interests. Such information allows one to begin to form a mental image of the people who comprise a department’s faculty. The make up of the student body, or *student demographics*, might be portrayed by data such as: How are students distributed by gender, age, ethnicity, etc.? How are students distributed over various categories, such as: mathematics majors; majors in various partner disciplines; part-time students; pre-service teachers; adults returning to school? What is student enrollment in the various courses?
Selected study indicators:

The following indicators have been selected for study, with data provided for each of the three pilot sites:

<table>
<thead>
<tr>
<th>Issue 0: Department Demographics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Key indicator 0.0:</strong> Characteristics of instructional staff and student body</td>
<td></td>
</tr>
<tr>
<td>0.1. Instructional staff by tenure status and gender</td>
<td></td>
</tr>
<tr>
<td>0.2. Instructional staff by tenure status and race/ethnicity</td>
<td></td>
</tr>
<tr>
<td>0.3. Student characteristics by enrollments in departmental course offerings</td>
<td></td>
</tr>
<tr>
<td>0.4. Student admissions policies (mathematics requirements for admission)</td>
<td></td>
</tr>
</tbody>
</table>

Issue 1 -- Department goals and priorities

(For details, see Chapter One below)

Any departmental self-assessment should surely be based on the department’s idea of its central mission(s). A mission statement can be useful in helping to formulate and concretize a department’s mission. Mission statements may at times be dismissed as collections of empty platitudes. However, this certainly need not be the case. They can, indeed, provide a focus for a department in terms of priorities and resource allocation.

Thoughtful mission statements will differ from one department and institution to the next. For example:

A mathematics department in comprehensive state university might have a significant mission of: “preparing K-12 teachers of mathematics.” Therefore, appropriate questions to ask would likely include: Are there mechanisms in place to facilitate communication and cooperation between the mathematics department and the institution’s school of education? How well are these mechanisms working, e.g., are the mathematics and education courses well coordinated? Is there good articulation between the department and the schools, e.g., does the department get useful feedback from the schools in which its graduates are placed concerning the content and the pedagogy of its courses?

In a community college setting, a departmental mission might include: “to provide adults in the community with opportunities to learn mathematics for job or career related purposes.” In such a case, one would probably want to study such questions as: How many adults in the community actually avail themselves of the opportunities provided by the department? Are the department’s courses offered at times and in locations convenient for the targeted audience of students? Do prospective students feel that these courses meet their needs? Is there coordination with, and input from, local employers?

On the other hand, in a major research university, one of the goals of undergraduate mathematics education might be: “to identify and groom prospective research mathematicians.” In such a setting, important questions to ask might include: Are there outreach programs to attract high achieving high school students to enroll in the department’s courses? Are there mentoring programs for advanced placement undergraduate majors, including, e.g., opportunities for students to participate in the research activities of individual faculty members? What proportion of the department’s undergraduate mathematics majors go on to graduate studies in mathematics or mathematics-related disciplines?
Selected study indicators:

**Issue 1: Department goals and priorities**

**Key Indicator 1.0: Departmental emphasis on undergraduate instruction**

1.1. FTEs committed to undergraduate instruction; graduate instruction; service and research
1.2. Instructional staff by tenure status teaching lower-division courses
1.3. Instruction takes place in flexible settings
1.4. Facilities are available to enable best possible teaching methods

**Issue 2 -- Program maintenance and monitoring**

(For details, see Chapter One below)

Departments make decisions about what courses to offer and about what content to put into those courses. There is a variety of issues involved in these basic decisions. Do departments make course decisions based primarily on the needs of the department, departmental goals, tradition and inertia, and/or other factors? Which are most important? Do departments have mechanisms that regularly re-consider courses and course content or is this done more informally? What do faculty think that the basis should be for these decisions? Do departments encourage instructors or department representatives to become and stay informed about broader national discussions of what should be done about and within various courses? What provisions are there in the department for review and revision of course offerings?

Selected study indicators:

**Issue 2: Program maintenance and monitoring**

**Key Indicator 2.0: Departmental provision for review and revision of course goals and content**

2.1. There is a written statement of department goals
2.2. A departmental syllabus exists for each course
2.3. Provisions are made in the department for the revision of courses
2.4. Course goals and content have changed within the past five years
2.5. Changes in course goals and content reflect recommendations of national reports
2.6. The department keeps abreast of the changing course needs of its students
2.7. The concerns of the department in choosing a text

**Issue 3 -- Professional development and instructional staff support**

(For details, see Chapter One below)

Almost all mathematics instructors finish their preparation "on the job" through various kinds of professional development opportunities that provide for career-long growth and change both in mathematics content and in mathematics pedagogy. Those opportunities can be provided on the local campus or externally at workshops, short courses, special meetings, etc. These can be specific to mathematics or they can cross disciplines (for example, a campus-wide seminar on effective instruction). Participation can be supported by departments in various ways ranging from informal encouragement to financial support and released time to official policies related to tenure, promotion, and salary.
### Selected study indicators:

<table>
<thead>
<tr>
<th>Issue 3: Professional development and instructional staff support</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Key indicator 3.0: Departmental support of instructional staff</strong></td>
</tr>
<tr>
<td>3.1. The department supports and encourages the professional development of its instructional staff in the teaching and learning of mathematics</td>
</tr>
<tr>
<td>3.2. The department supports and encourages the professional development of its instructional staff in the knowledge and ability to do mathematics</td>
</tr>
<tr>
<td>3.3. The instructional staff is using instructional technologies in the classroom</td>
</tr>
<tr>
<td>3.4. The instructional staff is using innovative instructional approaches in the classroom</td>
</tr>
<tr>
<td>3.5. The instructional staff participates in professional associations</td>
</tr>
</tbody>
</table>

### Issue 4 -- Partner disciplines

(for details, see Chapter One below)

Mathematics departments do not offer their courses in a vacuum. They serve a variety of student audiences and a number of partner disciplines. In particular, these include disciplines in science, engineering, and technology that make heavy use of mathematical tools. They include departments of education that are preparing teachers, both elementary and secondary, who will teach at least some mathematics. They include inducting those who will go on to further mathematics studies as mathematics majors and those who will go on to graduate studies in mathematics. They include students in business and social science areas who make frequent use of mathematical tools. Most widely, they include all students who need some preparation for mathematical literacy appropriate to a citizen in a technological society.

### Selected study indicator:

<table>
<thead>
<tr>
<th>Issue 4: Partner disciplines</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Key Indicator 4.0: Departmental communication with partner disciplines about student needs and program content</strong></td>
</tr>
<tr>
<td>4.1 The goals of the department speak to the needs and desires of a wide range of users of the mathematical sciences (that is, the ‘partner disciplines’)</td>
</tr>
</tbody>
</table>

### Issue 5 -- Instructional Strategies

(For details, see Chapter Two below)

Consistent concern has been expressed about whether the majority of lower division mathematics instruction makes use of varied and effective instructional methods. A number of methods beyond traditional lecture and whole-class discussion have been suggested as helpful: using calculators and computers, using small groups, using writing assignments, etc. These concerns raise questions. What are the varieties of instructional methods used for various types of courses? What is the balance between more traditional lecture-discussion methods and some of the more recently suggested alternatives? Do instructors feel they are using varied and effective instructional methods? How important do they think various methods are to effective instruction?
Issue 5: Instructional strategies

**Key indicator 5.0: Use of interactive teaching strategies**

5.1 Instructors use a variety of teaching strategies
5.2 Instructors use a variety of interactive (non-lecture) strategies
5.3 Instructors use strategies to promote student interaction
5.4 Instructors use a variety of mathematical representations while teaching
5.5 Instructors promote active engagement with mathematical content
5.6 Instructors are available to students outside of class

Issue 6 -- Classroom uses of technology

(For details, see Chapter Two below)

Computational, symbolic computational, and graphing technology have become more easily and widely available in recent years both for instructor demonstration in mathematics instruction and for students' hands-on experience. At the same time, this technology has changed much of mathematics or, at least, emphases in mathematics to include more applications and modeling. Also, these lead to development through new technology-based opportunities for visualization, etc.

This recent increased development of technology raises concerns about how this interacts with instruction. What aspects of technology are used in instruction for various courses? Do departments have policies or mechanisms that encourage or inhibit this use? How important do instructors feel that these tools are in lower division math instruction? What is their perception of how frequently they use these tools?

Issue 6: Classroom uses of technology

**Key indicator 6.0: Use of technology in the classroom**

6.1 Classrooms are equipped for using technology in instruction
6.2 Courses required by the department to use technology for instruction
6.3 Department provides support for technology for mathematics instruction
6.4 Department provides support for technology for mathematics research
6.5 Instructors use calculators in teaching
6.6 Instructors use technology (other than calculators) in teaching
6.7 Technology is used in teaching a variety of mathematics courses
6.8 Instructional staff considers themselves proficient in using technology for teaching purposes

Issue 7 -- Assessment

(For details, see Chapter Two below)

Assessment methods traditionally have been by paper and pencil tests, quizzes, and/or homework. More recently, alternative forms of assessment have been suggested: projects, writing assignments, collections of student work, etc. Further, there is concern whether this assessment is used merely to grade students (summative evaluation) or also to help them learn more effectively (formative evaluation). Formative evaluation can inform and direct learning by helping diagnose what students are doing and how they might refocus their efforts. A number of questions follow from these concerns. What assessment methods are actually used in various types of courses? How frequently are these methods used and/or emphasized? How important do faculty feel that these various assessment options are? Which, if any, are used for feedback and "formative" evaluation of student progress? Which are used in final "summative" grading?
Issue 7: Assessment

**Key Indicator 7.0: Use of assessment methods**

- 7.1 Instructors use a variety of assessment methods
- 7.2 Instructors seek student feedback to monitor progress
- 7.3 Instructors use a variety of criteria in determining final grades
- 7.4 Instructors assess core student proficiencies using common items

Issue 8—Student Retention

(For details, see Chapter Three below)

A perennial issue for college mathematics instruction is the high failure and drop out rates of many courses. The issues are varied. What are the failure and withdrawal rates for key lower division mathematics courses? Are these rates consistent over time? Do they affect certain kinds of students more than others (e.g., minorities, those returning to school, part-time students, etc.)? Are there specific programs in place designed to increase retention and success? How do students perceive mathematics courses -- as filters and gateways or as something more positive? Do students perceive departments and instructors as concerned, active in helping them succeed, and/or effective in doing so?

The issues of retention relate the recruitment of majors in the areas of science, mathematics, engineering, and technology education. This also includes education-related majors in these specialty areas. Another issue related to retention is the issue of remediation. How do these two issues and their associated statistical indicators correlate?

Issue 8: Student retention

**Key Indicator 8.0: Student intention to continue in the study of mathematics**

- 8.1 Students feel that the instructor is aware of mathematical needs of their major field of study
- 8.2 Students believe that the content of the course they have just completed will be useful in their future
- 8.3 Students look forward to taking more mathematics
- 8.4 Proportionate numbers of women and ethnic minorities intend to continue in the study of mathematics.

Issue 9 -- The mathematics department as a community of learners

(For details, see Chapter Three below)

Students studying mathematics may often feel disconnected from anything other than the specific course that they attend, a course in which they may often feel like they are just one more anonymous “warm body.” Many have suggested that this isolation affects the quality of mathematics learning. Thus, deliberate efforts to involve students in the life of the mathematics department, with instructors, and other students outside of class can improve their learning. A variety of questions emerge. Do students feel isolated? Do they feel that it affects their learning or do they think it's the “way it's supposed to be”? What efforts, formal and informal, do departments and instructors make to involve their course students (as opposed to majors) in the department's life, in contact with instructors outside of class (both in office hours and other ways), and in contact with other students studying mathematics but outside of their classes?

To some extent, the level of student involvement in the life of the department begins with engagement of the student in the classroom. Are students motivated by their study of mathematics in the current classroom? Are students encouraged to interact with other students in classroom activities? How about outside of class time? Are students encouraged and offered the opportunity to interact with the instructor both inside the course and outside? If students are engaged by the study of mathematics and motivated to interact with each other, then they are more likely to be interested in the life of the department outside of the framework of their course.
Selected study indicators:

<table>
<thead>
<tr>
<th>Issue 9: The mathematics department as a community of learners</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Key Indicator 9.0: Student participation in the life of the department</strong></td>
</tr>
<tr>
<td>9.1 Students take part in supplementary (non-class) mathematical support services</td>
</tr>
<tr>
<td>9.2 Students take part in supplementary (non-class) mathematical activities (e.g. lectures, colloquia, math clubs, etc.)</td>
</tr>
<tr>
<td>9.3 Students feel that technology (calculators and/or computers) is helpful in learning mathematics</td>
</tr>
<tr>
<td>9.4 Students take part in social activities of the department</td>
</tr>
</tbody>
</table>

**Issue 10—Diversity**
(For details, see Chapter Three below)

College mathematics has been characterized as more often serving white middle and upper class males. Various mandates and programs have been suggested to try to remedy this situation. They raise a number of issues. How well does participation in various types of mathematics courses reflect the gender, racial, and socio-economic make up of the general US population? of the college-going population? Do some courses have more "under representation" than others? Do some courses "over represent" under-represented student types? Are there specific programs and practices in place to address these issues? Are these practices and programs mandated from above the department level, at the department level, or do they flow from widely shared concerns about these issues? Do students, both those underrepresented and the more general population of students feel that there is concern for these issues?

**Key Indicator 10.0: Proportional representation by gender and race of students in mathematics courses**

| 10.1 (Lower division) student enrollment in the department shows gender diversity |
| 10.2 (Lower division) student enrollment in the department shows ethnic diversity |

**0.8 The AERA Grants Board Indicators Project**

A forerunner to the current project was an initiative funded by the American Educational Research Association (AERA), through a grant from the National Science Foundation, in January 1994. The charge to the AERA Project was as follows:

The focus ... is to be on undergraduate mathematics education indicators. Concern is to be primarily with lower division programs for the entire population of students, not just those majoring in mathematics. Concern is also to be for the broad spectrum of public and private institutions including community colleges, liberal arts colleges, comprehensive universities and research universities.

The Project was to address issues such as:

a) What aspects of undergraduate mathematics education can and/or should be monitored by an indicator system? In this regard, a broad view of mathematics will be taken. Conceptual understanding, literacy, sense making and opinions and attitudes will be included for consideration.

b) What implications of indicator systems need to be taken into account, including the possible unintended and undesirable effects of monitoring mechanisms?

A major product of the AERA Grants Board Project was a set of papers addressing key issues in undergraduate mathematics education, as included in Volume Two below. Titles (and authors) and brief descriptions of the papers are as follows:
a) Curriculum and instruction (Curtis C. McKnight, University of Oklahoma)

Indicators of curriculum and instruction are the measures of educational experiences and the mediators of student accomplishment. This dual responsibility -- namely, stand alone characterizations of students' experiences in mathematics education and interpretive contexts for assessments of students' accomplishments -- is both an obvious and essential argument for the inherent importance of these indicators.

b) Assessment of student outcomes (Alan Schoenfeld, University of California, Berkeley and John A. Dossey, Illinois State University)

Indicators of undergraduate mathematics performance are to emphasize the broad spectrum of mathematics necessary for a technologically and mathematically sophisticated work force, as well as the kind of substantial mathematical background(s) that will enable increasing numbers of American students to go on to careers in the mathematical and other sciences.

c) Student participation (Sylvia Hurtado, University of Michigan and Eric Dey, University of Michigan)

Indicators to provide data on student participation and persistence in mathematics and mathematics-based careers are from the following categories of information: Pre-college preparation of students, Transition and articulation, Undergraduate programs, Assessment and evaluation, Institutional characteristics, and Public attitudes, understanding and support.

d) Institutions and departments (Peter Ewell, National Center for Higher Education Management Systems with the assistance of Janet Ray, Seattle Community College)

Institutional indicators monitor a broad range of issues that reflect the context within which the teaching and learning of mathematics occurs. It is important to recognize the diversity of settings in which this takes place, from Ph. D. granting research universities to two-year community colleges. Qualitative as well as quantitative data are needed to adequately portray the various aspects of these diverse contexts.