Abstract

A Framework for Monitoring and Increasing Undergraduate Student Participation in Mathematics Education

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This chapter presents information regarding the changing nature of the student population in higher education and the implications that these trends have for student participation in undergraduate mathematics. Given present concerns with issues of equity in terms of access and participation, it will continue to be important to collect indicator information for women and various racial/ethnic groups while also considering the need to do so for emerging groups central to policy interest (e.g., re-entry students, teachers in training).

In order to expand the current collection of traditional pipeline indicators, a conceptual framework is presented to help organize categories of proposed indicators that influence student decisions to participate in undergraduate mathematics. These categories of indicators include the general social environment and institutional practices; student cognitive and affective orientations regarding mathematics; current formal and informal types of student involvement in the study of mathematics; and student participation in light of changing practices in undergraduate mathematics. Subcategories of indicators within each of these areas, along with sample indicators and potential sources of data are also presented. In the framework, student participation in mathematics is broadly construed in order to reflect that students enter, leave, and re-enter higher education at various life stages and with increasing frequency. This suggests that students may leave only to need to become re-acquainted with mathematical concepts and skills at subsequent points in their academic careers.
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1.0 Introduction and overview

The traditional ‘educational pipeline’ metaphor can be a useful way of describing the problems and hierarchical nature of educational progress in mathematics, but it is a metaphor which is not ideal in several respects. To begin, it is important to note that the pipeline perspective is lacking in its ability to suggest alternative strategies designed to increase student participation in mathematics. Due to the collection of pipeline indicators, for example, data have long been available which suggest that men and women are not equally likely to earn mathematics degrees, nor are students from various racial/ethnic groups. This information is useful in pointing to student populations and areas needing further attention, but it does not suggest how we might go about increasing access and equity in undergraduate mathematics education.

A second limitation is that simple pipeline indicators traditionally focus only upon courses or major fields of study offered through mathematics departments, which serves to reinforce the notion that student learning of mathematical concepts and skills is confined to a limited number of academic fields. Such a perspective then can only offer an imprecise view of the status of mathematics education because it obscures the reality that a good deal of quantitative reasoning and mathematical literacy courses -- including economics and statistics courses in applied and professional fields -- are offered by a variety of academic departments.
One estimate, for example, suggests that the majority of enrollments in courses which emphasize advanced mathematics occur outside of mathematics departments (Garfunkle & Young, 1990).

A third limitation is that the pipeline metaphor and its indicators are most appropriate for understanding the progress of traditional college-age students. Perhaps with the exception of colleges that continue to recruit and enroll first-time, full-time freshman students, colleges and universities are now serving students from a variety of backgrounds and experiences. These changes are most obvious in terms of demographic characteristics, but there have also been sharp changes in the preferences students express for different major fields and post-college careers; both of these trends have required responses from institutions of higher education interested in accommodating students (Dey & Hurtado, 1994). One of these changes is related to increased access, where relative to the past there are many additional paths students follow in pursuing a college education. Moreover, it is no longer the case that once a student leaves college that he or she remains out of the educational pipeline, as students depart and re-enter the higher education system with surprising frequency. These multiple points of access to and departure from college, coupled with the movement of students in and out of the fields most directly related to mathematics, further limit the utility of the pipeline metaphor.

If important aspects of the progress of students through mathematics education are not captured by the pipeline metaphor, what might? Our view is that a more appropriate metaphor for undergraduate mathematics education is a metropolitan transit system. In contrast to a simple pipeline, a transit system offers many different points of entry and departure which passengers can combine in order to reach their desired destination (a goal which can change several times enroute). Passengers can choose different routes to reach their goals, and can also choose different modes of transport -- personal vehicles, taxis, express trains, buses, or they may chose to walk -- to negotiate these routes depending upon their needs, preferences, and resources. The diversity of higher education settings provide students with similar kinds of institutional options for reaching their educational destinations, with the array of mathematics education available
throughout the undergraduate curriculum providing even more options. In addition to these structural considerations, the transit metaphor also captures aspects of the student experience. For example, students experience the same classes in different ways -- just like the experience of a bus ride is influenced by factors like the expertise of the bus driver, the number and nature of other passengers, and things happening outside of the bus. And like commuting passengers facing daily transportation decisions, students make course choices on a regular and continuing basis, with each choice exerting a degree of influence on subsequent ones due to curricular offerings and prerequisite structures.

In this chapter, our objective is to develop a framework for enhancing our capacity for monitoring undergraduate mathematics participation -- broadly defined -- in order to suggest ways to increase the quantitative literacy of all undergraduates, as well as foster talent among those students interested in pursuing studies most directly related to the field of mathematics. Toward this end, we have developed a simple conceptual framework which is intended to capture the most important aspects of student participation in mathematics. (In developing the framework we will emphasize those aspects of undergraduate mathematics participation highlighted by the transit system metaphor introduced above, but will not dwell upon the metaphor itself.)

After briefly reviewing changes in the characteristics of students that are most relevant to participation in undergraduate mathematics, we introduce the framework which focuses on the intention and transaction components of the model presented in Chapter 0 and which highlights the processes students use to make choices with respect to future participation in mathematics. Nested within the larger social and educational context, it is our belief that this framework will prove useful because it focuses attention on the social and psychological indicators that play a role in determining student choices relative to mathematics. Indicators based on this framework, in combination with efforts to monitor access and equity issues among gender and racial/ethnic groups, should be useful in suggesting future policy initiatives designed to increase student participation in undergraduate mathematics.
2.0 The changing context of undergraduate education

2.1 Student characteristics and participation patterns

Over the past several decades there have been major changes in the composition of American college students along a variety of dimensions. In terms of traditional demographic characteristics, there have been sharp increases in the number of older students, women, and racial/ethnic minorities represented at the undergraduate level (Carter & Wilson, 1992; Dey & Hurtado, 1994). There has been a reversal of a historical pattern of gender under-representation within higher education. Women now represent the majority of students in higher education in general, as well as constitute the majority of those entering as first-time, full-time freshmen (Solomon, 1985; Dey & Hurtado, 1994).

Enrollment patterns have also been changing. Adults over the age of 25 have been the fastest growing group, currently representing over 40 percent of all students in higher education (U.S. Department of Education, 1992). There has also been a shift toward part-time enrollment in higher education since 1965, with part-time students now representing about 43 percent of all students. Not unrelated to these increases in part-time attendance has been a gradual increase in the number of years that undergraduates take to complete a baccalaureate. Less than 40 percent of students who begin full-time study in higher education finish in four years. Although the reasons for this change are not entirely clear, it implies that students may be stopping out of college, working long hours to pay for college, and experiencing difficulty in completing requirements for the baccalaureate. This extended time to degree may have as much to do with the structural constraints in institutions (e.g. sufficient FTE allotments to offer required courses at convenient times) as it has to do with the types of students now pursuing higher education. Nevertheless, all of these events preclude consistent student involvement in the classroom and mathematics-related activities outside of the classroom that might enhance students’ quantitative reasoning skills.
In addition to the changing composition of students, there also appear to have been changes in student preparation. The Cooperative Institutional Research Program (CIRP) surveys of first-time, full-time freshmen show that students now are entering college with higher grade point averages than in the past (Dey & Hurtado, 1994). The percentage of students earning A grades in high school essentially doubled between 1966 and 1993 (from 15 to 27 percent, respectively), while the percentage earning C or worse grades fell by one-half (31 to 16 percent). The relatively high level of student academic success before college appears to have influenced the expectations that students have for college: These same data show strong increases between 1973 and 1993 in the percentage of entering students who expect to earn at least a B average in college, graduate with honors, and be elected to an academic honor society. Over the same period, the percentage of students expecting to fail one or more courses dropped by one-half. In combination, these patterns might suggest that students today have come to expect better grades for their performance than those entering college two decades ago.

At the same time, these data also reveal a contrasting perspective. In spite of their apparent academic success in high school, students also report that they are in need of additional academic support services. For example, the percentage of students who believe that there is a ‘very good chance’ that they will need tutoring help in specific courses during college more than doubled between 1975 and 1992, from 7 to 16 percent. Similarly, the percentage of students who expected to get special tutoring or remediation in mathematics, science, and foreign language increased between 1984 and 1993. This is especially interesting since, over the same period, students were more likely to meet recommended levels of preparation in many of these fields. For example, more than 93 percent of students who entered college in 1993 had at least three years of mathematics during high school (up from 85 percent in 1984), while three out of every ten students expected to need special tutoring or remediation in mathematics (up from 22 percent in 1982).
In combination, these trends suggest that the relationship between academic course work on the high school level and proficiency related to those courses has been redefined. This puts tremendous pressure on college and university faculty to work with students who have been academically successful in high school and have met or exceeded recommended levels of high school study, but who may nevertheless be under-prepared for college-level work. Unfortunately, many new college entrants are not aware of the extent to which they must compensate for inadequate mathematical skills until they are faced with initial results from college placement tests or experience difficulties in the classroom during the first year of college. The issue of pre-college mathematics preparation and experiences affects all institutions. While improvements are implemented in K-12 mathematics education, community colleges and four-year institutions that attract a broad clientele face the biggest challenge of meeting a diverse set of student needs in the study of mathematics.

Faculty and policymakers will continue to face formidable challenges in engaging and maintaining student interest in mathematics. Student preferences and interests have generally shifted away from the traditional liberal arts fields toward professional education, with perhaps some of the best-prepared students seeking majors and careers in business, health-related fields, and education. Between 1967 and 1992, there was an 85 percent decline in the number of entering students interested in mathematics and statistics as a major field of study (Dey & Hurtado, 1994). Although the new and developing field of computer science may have captured some of the students who were interested in mathematics as a field of study, it is more likely that the decline is due to a shift in student interest toward professional schools. Students identified with high mathematical proficiency may actually never take a course in a mathematics department, particularly if they place out of institutional requirements and/or take quantitative reasoning courses necessary for their major in professional schools or other departments. Thus, cultivating mathematics majors among students who have excelled in mathematics prior to
college is made more difficult by the prevailing norms of student interest and beliefs about mathematics.

The implications of these and related trends are important for monitoring changes in mathematics participation among students. Taken together, they suggest that we have an increasingly fluid population of students who may participate in the study of undergraduate mathematics at different life stages; at different stages in their undergraduate education; with different attitudes, interests and motivations; and with very different preparation and proficiency levels. These trends suggest that assessing student skills in mathematics as part of college placement efforts becomes especially important. In addition, facilitating articulation at major transition points in the educational careers of students -- from high school to college, between mathematics courses within a particular curriculum or institution, and between different sectors of higher education -- increases in importance as students may enter the system at many different levels. Moreover, it requires that we strive to monitor not only those students who follow more traditional routes toward advanced degrees in mathematics, but that we also monitor the educational experiences of those in other areas. The goal of improving quantitative literacy suggests that rather than simply concentrating on the most mathematically gifted, that we need to consider mathematics participation at all ability levels and in all sectors of higher education. Finally, because of the changing demographics of many institutions and states, it is important to continue to monitor the representation of women and racial/ethnic minority groups participating in all types of mathematical experiences.

2.2 Access and equity in mathematics

Specific populations of students require continued monitoring with regard to specific problems encountered with participation at various levels of the study of mathematics, degree attainments, and representation in mathematics-related careers. Women and racial/ethnic minorities are expected to make up an increasingly large proportion of the nation’s workforce, and their numbers have already increased substantially at the secondary and postsecondary levels
of education. Each group, however, faces different problems with regard to their participation in higher education as well as confront different issues specific to mathematics participation. At the same time, newly identified subpopulations of students may require additional attention in the future. For example, monitoring the patterns of participation of part-time students in mathematics may present new challenges for institutions, including the necessity of providing a wide range of course offerings each academic term to ensure that part-time students can progress through various course sequences. Older and re-entry students’ mathematics participation may be monitored to determine how institutions and students deal with the ‘re-acquaintance’ process of overcoming mathematics anxiety compounded with anxieties regarding overall student performance relative to traditional college-age students. While the universal language of mathematical symbols may be appealing to language minority students, they may be hindered by specific types of mathematics assignments that require English proficiency. Although these concerns are not unique to mathematics courses, overcoming these problems may be important determinants of student success in the classroom and choices regarding subsequent coursework. Another population that is important to monitor are teachers in training, as the attitudes and preparation of our future teachers are expected to impact future generations of students eligible to participate in mathematics at the postsecondary level.

2.3 Guiding questions for understanding student participation

In order to identify the needs of various student populations, as well as increase the number and diversity of students who pursue mathematics-related careers in the future, we seek data that will help answer the following questions regarding the interaction of student characteristics and participation in mathematics:

- Who participates in various forms of mathematics or quantitative reasoning education?
  Which groups are more likely to begin at particular levels of remedial, college-level, and advanced mathematics?

- How do students of various gender, racial/ethnic groups and ability levels fare with regard to indicators in the framework of factors that impact mathematics participation?
Are particular factors more likely to be deterrents or facilitators for specific groups?

Which groups benefit most (or least) from changing educational practices? Which groups show the most improvement in continuous enrollment of various curricular sequences? Are particular groups systematically excluded from participation in curricular innovations because they are either not available at their college or not offered for their proficiency level (e.g. students in remedial courses, disabled students, etc.).

At what transition points are students from various ethnic/racial, gender, and ability groups most likely to cease taking mathematics courses?

Which groups are most likely to cease or resume taking mathematics at later periods in their lifetime? What institutional supports facilitate “re-entry” into mathematics?

Monitoring student populations where the problems of quantitative literacy seem most intractable may produce new insights into how students learn and lead to innovations in teaching that will increase overall undergraduate participation in mathematics. However, this is most appropriately accomplished by also monitoring some of the larger institutional and social context factors surrounding student participation in mathematics education. These are reflected in the framework presented in the next section.

3.0 A general framework for monitoring undergraduate participation in mathematics

In order to organize indicators that reflect the many aspects that play a role in student choices relative to mathematics education, we present a conceptual framework drawn, in part, from the work of Lantz (1985) who sought to identify strategies for increasing mathematics enrollments among women. Other studies have used elements of the framework presented here. For example, Eccles and Jacobs (1986) emphasize the social influences that affect student self-concept, perceptions, and math anxiety in relation to future enrollment plans in mathematics courses. McLeod (1994) emphasizes the importance of research on attitudes, student beliefs, and emotional responses to mathematics in relation to the study of mathematics. The Longitudinal Study of American Youth also utilizes many measures of the social and psychological influences presented here in studies of student persistence and success in mathematics (Miller, 1992).
The framework, shown in Figure 1, is based on the assumption that a number of factors play important roles in helping to determine the choices that students make in terms of their future participation in mathematics. We should note at the outset that this framework is used primarily as an organizational device that emphasizes both intentions and transactions, but which is not meant to be a comprehensive model. Major elements of the intentions-transactions-outcomes model are addressed in depth in other chapters within this volume (e.g., assessment of mathematics proficiency in Chapter 2). In this chapter we will focus primarily on developing indicators related to social and psychological factors while paying special attention to issues of access and equity with respect to mathematics participation.

It is important to first note that the framework depicted in Figure 1 represents a cyclical process in which intentions and educational transactions play important roles. Students make decisions about their participation in mathematics on a regular basis, with these cycles tied to the academic calendar. In short, we need to be able to monitor a series of student choices. In addition to these regular cycles, there are several transition points within most undergraduate programs which merit special attention since some of these transition points usually correspond to those typically addressed by pipeline indicator systems (and presumably where most students make critical decisions regarding their future participation). These include the transition from high school to college (or upon entry for those not proceeding directly from high school), the college placement process, the transition from remedial to college-level mathematics courses, the point at which students formally declare their major field of study (which in many mathematics-related fields now occurs as early as the first year of college), and upon leaving the undergraduate program (either through early departure via withdrawal, or by progressing into post-graduate education or into employment).

Represented at the top of this cyclical framework is a broad grouping of factors related to the social environment and educational practices. As used here, the social environment encompasses norms and expectations that exist in general society and within institutions. With
respect to general social factors, it may be that social norms and expectations may differentially influence certain groups of individuals to seek out (or avoid) participation in the study of mathematics, or that job market forces associated with mathematics-related career fields may serve to track students into different types of mathematical experiences. In addition to general social norms that affect our understanding of who can or should do mathematics, students are influenced by parent and peer orientations that provide support or discouragement in pursuit of mathematics and mathematics-related fields. Social norms and expectations operate within institutions as well, and are expressed -- in part -- by the educational practices which are favored in different institutions. Faculty beliefs and expectations are one aspect of this, and are expressed in practical terms through classroom climate and providing a variety of opportunities (such as mentoring, research involvement, etc.) to different groups of students. Such forces can also generate an institutional climate which fosters curricular and instructional innovation (see Chapter 4), while differential curricular emphases and institutional requirements are examples of ways in which educational practices significantly influence student participation in mathematics education.

The primary group of indicators to be considered in the framework depicted in Figure 1 are those factors directly related to students and their personal interests, proficiency, and identity with respect to mathematics. In addition to previous participation in mathematics, student factors can be divided into three major areas: Cognitive beliefs and aspirations, affective orientations, and mathematical proficiency. Following Lantz (1985), the cognitive beliefs element within this group encompasses “various beliefs about the usefulness or utility of mathematics for the individual’s educational and career aspirations” (p. 331). In contrast, affective orientations encompass not only student preferences, but also student self-confidence, mathematics anxiety, and enjoyment/challenge experienced relative to mathematics. Mathematical identity is a construct which is related to each of the other elements within this group of student factors, and is intended to capture the degree to which students view themselves as "becoming"
mathematicians (or as individuals with the ability to do mathematics in its various forms as a professional). Mathematical proficiency encompasses a variety of indicators including not only assessments of different types of mathematical skills but also achievement measures (such as test scores, assessments of course performance, and other indicators detailed in Chapter 2). For clearly, proficiency and assessments of performance determine further participation in course sequences. Not only do students seek majors and careers in which they feel competent and self-confident, but it is also true that they cannot enter particular mathematics-related majors or fields unless they are judged by others to be competent.

The final group of factors assumed to play a role in determining future mathematics participation is current mathematics participation, for these experiences help direct students choices in subsequent decision cycles. This group of indicators encompasses a broad definition of participation experiences that include, but go beyond pipeline indicators such as degree attainments. The type of course in which students are exposed to mathematics is an important consideration, as is the degree to which the student is participating (or has participated) in supplemental experiences or parallel programs designed around mathematics. Examples of the latter include academic and social opportunities to build mathematics skills outside of course enrollments, including special summer programs, mathematics clubs, research projects, and internships in mathematics-related careers. A final element within this group is a broad one, and is intended to capture the nature of the student experiences within the courses they are taking. Was the student engaged in classroom activities or required to be a passive? How successful was the instructor in motivating the student? What was the classroom climate, and did the student perceive it to be both supportive and challenging? Was the student able to perform at a level consistent with his or her goals and expectations? These considerations are important, especially in regard to issues of access and equity.

All of the factors mentioned above are assumed to be related to student participation in mathematics occur within the context of changing educational practices (referenced in Chapter
1). Institutions and faculty are constantly striving to meet student needs and to respond to external pressures for student performance, while also applying the benefits of pedagogical innovations. These developments serve, in turn, to influence student experiences in each cycle (which then influence student decisions for participation in subsequent cycles). The implication of this dynamic system of interrelationships is that any system designed to monitor student participation in mathematics education should be structured to measure indicators on a regular basis in order to capture changes, patterns, and identify areas for future policy development.

4.0 Indicators of participation and factors that influence participation

4.1 Social environment and educational practices

This group consists of categories of indicators that include social norms, institutional requirements, faculty expectations, and perceptions/knowledge of the market value of careers associated with mathematical proficiency. Each of these categories is explained more fully in the sections that follow, and each is accompanied by examples of indicators or data that might be used as sources of information.

4.1.1 Social norms continue to largely determine expectations and stereotypes regarding who is a mathematician, what personal qualities are required to do mathematics, and whether particular students that start out with lower mathematics skills can be taught to reach acceptable proficiency levels to perform particular jobs. That is, there is still a prevailing sentiment that ability to do mathematics is genetic and one must possess a ‘math gene’ in order to do well or to pursue a mathematics-related career. However, the notion of quantitative literacy suggests otherwise: Both essential and sophisticated mathematical concepts and skills can be learned and are necessary for all students to function in today’s society. This view of the importance of widespread mathematics participation has not permeated the thinking of the American public, however. Because public sentiment ultimately influences policy, indicators of general public sentiment might come from a national public opinion survey (such as the General Social Survey) that can tap into support for mathematics-related endeavors, beliefs regarding the
importance of learning advanced levels of mathematics, and changing social conceptions of
groups (women, minorities, older students, etc.) and their ability to do mathematics or enter
mathematics-related fields.

Social norms include indicators of general public perceptions regarding the importance of
mathematics literacy, but they also include parental expectations and peer norms. Parents exert
influence on students throughout their schooling. Many studies have shown how parental
educational attainments and orientations often directly or indirectly affect college students' choice
of career and choice of careers considered to be atypical for their sex (Pascarella & Terenzini, 1991). In addition, research at the precollege level has clearly established a link between parental
expectations and encouragement to persist in mathematics (Miller, 1992), particularly among
young women (Eccles & Jacobs, 1986). In addition to numerous local studies conducted in
higher education and in the K-12 literature, at least two national longitudinal surveys monitor
parental influence using measures that capture either students' perceptions of parental
encouragement or direct measures of parental expectations. These are the Longitudinal Study of
American Youth (LSAY) and the National Educational Longitudinal Study of Students
(NELS:88). The NELS specifically obtains data directly from parents, suggesting that parental
orientations can be linked with student participation and persistence in mathematics. While these
measures have been designed to be used in connection with other primary indicators of student
persistence and achievement, there is no reason why parental orientations cannot be monitored as
'stand alone' indicators of public perceptions of mathematics that are more proximal in their
impact on student participation in mathematics.

In addition, peer norms are powerful influences with regards to participation in a wide
range of social and academic activities and outcomes (Astin, 1993). Institutions provide
supportive peer environments when they have a large number of majors in mathematics or
students pursuing mathematics-related majors, provide opportunities for peer groups to form
surrounding challenging mathematics assignments (Treisman, 1992), and provide opportunities
for students to teach what they know to other students (Whitman, 1988). The extent to which institutions are creating a climate for involvement in mathematics might be monitored through surveys similar to that devised by the Mathematics Association of America. Sample indicators: *Institutional or departmental, use of undergraduate teaching assistants and tutors in mathematics; faculty pedagogical styles that encourage interaction, working groups, cooperative learning activities in quantitative reasoning courses; student reports of peer encouragement to pursue mathematics coursework or mathematics-related majors/careers.*

4.1.2 **Market value of mathematics careers and mathematics-related careers** may influence students’ initial preferences for specific careers and majors, so long as they remain aware of how important participation in mathematics is to careers where there is future job growth. Indicators of the market value of mathematics and mathematics-related careers include *monitoring starting salaries, personal income over the lifetime of individuals taking into account the investment of a college education, and forecasting developing technologies and expected areas of job growth that require quantitative reasoning skills.* Most of these indicators are already available as part of the typical collection of information from the Bureau of Labor Statistics. Monitoring the impact of campus or departmental 'career awareness' activities may determine the extent to which information (or lack of information) influences student decisions as they select courses or proceed through transition points in their undergraduate career. Sample indicators: *The extent to which students are aware of market value; ways in which institutions' career planning or undergraduate advisors communicate this to potential majors; the extent to which institutions make information about alumni to their students.*

4.1.3 **Institutional requirements** largely influence student decisions to take mathematics courses. Some colleges pride themselves on the lack of requirements, others have standard general education requirements that include mathematics or quantitative reasoning requirements, and still others offer mathematics alternatives and many options to meet distribution requirements. Some institutions have different sets of requirements for business,
nursing, engineering, and education school undergraduates that differ from the rest of the institution’s general education requirements for students in other fields. The type of mathematics or quantitative reasoning requirements, or lack thereof, may largely determine the impact of different types of institutions on students’ quantitative reasoning skills, course enrollments, and ultimately, the number of majors and degrees awarded in mathematics and mathematics-related fields. Thus, participation rates should be examined, taking into account these curricular requirements. Institutions that allow substitutions for mathematics in distribution requirements, allow students to ‘place out’ of mathematics requirements, or have no specific requirements for quantitative reasoning development in their general education curricula are likely to have lower course enrollments (and perhaps fewer mathematics majors) even if they tend to recruit very talented students. Furthermore, there are some who feel that strict adherence to prerequisites deters students from pursuing further mathematics or mathematics-related careers (Tobias, 1990), while others contend that a loose prerequisite structure at an institution hinders students from obtaining the proper education they need to build upon mathematical concepts and skills. Clearly further research is needed in terms of understanding the link between institutional requirements, adherence to the prerequisite structure, and faculty advising activities to determine their impact on course enrollment and individual persistence in mathematics through the major. Institutional surveys of mathematics departments, similar to the one distributed by the Mathematics Association of America, would be able to capture institutional requirements, adherence to a prerequisite structure, and advising activities. However, in order to understand overall institutional contributions to quantitative literacy, a more comprehensive survey of institutional requirements or the inclusion of many more departments that offer such coursework may be necessary than currently exist.

4.1.4 Faculty expectations for students who enter college with differing skills and abilities from different social backgrounds has not been examined. However, the effect of teacher expectations on students at the precollege level has generated a great deal of evidence that
suggests that students often rise to the level of teacher expectations. Although the impact of expectations have not been examined at the college level, primarily because it requires linking data on student outcomes to data on faculty attitudes towards students in their classrooms, surveys of college faculty in the late 1980s have shown that about 60 percent are satisfied with the quality of undergraduates (National Science Foundation, 1992) but only about 27 percent think that their students are well-prepared academically (Astin, Korn, & Dey, 1991). Indicators of faculty satisfaction with student quality and expectations for student success in mathematics and mathematics-related departments, collected through attitudinal surveys of faculty, may be linked with student decisions to participate as well as participation indicators at the institutional level that are relatively easy to monitor (e.g., degree attainments, course enrollments, etc.). Periodic surveys of faculty administered by NCES (the National Survey of Postsecondary Faculty) and/or the Higher Education Research Institute might be used to collect such indicators. Current national surveys of college students administered by both these survey sponsors could also include indicators of student perceptions of faculty expectations. Of particular importance from the perspective of students is the extent to which they feel valued, welcomed, and validated by faculty and other members of the campus community (Hurtado, 1992; Terenzini et al, 1994; Turner, 1994).

4.2 Student decisions relative to participation

This construct suggests both the timing of data collection as well as specific indicators of student decisions to pursue additional mathematics education. The appropriate time for taking stock of indicators might be key to developing a better understanding of student decisions, and presumably ideal times for implementing intervention strategies. If course choices are dictated by the academic calendar and occur on a cyclical basis during registration periods for the following term, it might be appropriate to assess many of the student cognitive and affective indicators during this period (e.g., student aspirations and interests, affective orientations, beliefs about utility, etc.), as well as collection of indicators of previous experiences that include judgments.
about courses and instructors, reasons for maintaining (or losing) an interest in mathematics. Transition points are key periods for assessment of many of the student level indicators. While this information is derived directly from the student regarding his/her beliefs and concerns, it is also important to note the institutions’ role in determining students’ decisions.

4.2.1 Course choices according to the academic calendar, as indicated by the framework, are influenced by immediate experiences regarding participation in mathematics as well as cognitive and affective orientations that preceded participation. However, some choices that are immediately affected by the academic calendar may be determined by examining: The term and specific course in a students’ academic career when they commonly cease to take additional mathematics courses; students’ choice (or perceptions of lack of choice) of available and appropriate mathematics courses in the next term; students’ view of convenient time of day, instructor reputation, or format of mathematics courses available next term; reports of timely receipt of advising/counseling as to next appropriate mathematics course to take for the student’s career interest or major field; qualitative accounts of the factors that lead to decisions not to continue with course sequencing; and, student evaluations of instructors of courses just completed, controlling for performance in the course and whether the course was required or elective.

4.2.2 Important transition points reflect key times in which student decisions take place. These periods of student decision making are influenced by many aspects of the model, but here we focus on a few indicators having to do with a variety of student behaviors and institutional practices which may be examined at transitions between high school and college, between sectors of higher education, and between courses within institutions.

Transition from high school to college. This transition captures the extent to which students continue to take mathematics in college after meeting high school graduation and college entrance requirements. It also includes the extent to which mathematical talent identified in high school is cultivated in colleges through
encouragement to take additional course work in mathematics (rather than “placing out” of requirements through AP examinations), becoming involved in honors mathematics course work, participating in research with faculty, etc. during the first year of college. Sample indicators: Number of entering students who continue to take mathematics in college at all proficiency levels; Proportion of entering students who state they need additional tutoring or remedial work to build their skills in college; Proportion of entering students who fulfill mathematics requirements prior to college and continue (or do not continue) with advanced levels of mathematics; Proportion of entering students who begin honors mathematics or participate in mathematics-related projects with faculty. The CIRP offers several indicators of entering student concerns regarding their preparation for college. The Beginning Postsecondary Survey (BPS) of students, administered by NCES, could be expanded to include more extensive indicators of this transition from high school to college with special emphasis on mathematics. Currently there are very few items about relevant high school experiences and transition information in the BPS, and yet, expanding this would be relatively easy in the next round of survey development and planning for longitudinal cohort studies. Subsequent transitions detailed here can also be developed in future BPS follow-up surveys.

**College placement.** Monitoring this critical transition activity includes assessment of the appropriateness and utility of college placement examinations in the transition from high school to college, or from two and four year institutions. Sample indicators: Survey-based measures of students’ view of appropriate placement according to their proficiency levels; faculty views of appropriate placement of students in their classes; and departmental views of the utility of the variety of placement exams currently in use.

**Movement out of remedial tracks.** The movement of students from remedial to college level course work in mathematics or other subjects requiring quantitative
reasoning. Sample indicators: *Number of students who take remedial course work and enroll in precalculus or calculus; number of students who move from remedial mathematics to elect mathematics-related majors; number of institutions that require or provide incentives for movement from remedial course work to mathematics-related subjects.* These indicators can be gathered through longitudinal surveys of students and/or data on student coursework from student transcripts can be directly obtained from institutions.

**Transfer of coursework and articulation between institutions.** This includes monitoring the movement of students from 2 year to 4 year colleges in mathematics and mathematics-related fields. Sample indicators: *Institutional reports of mathematics and mathematics-related majors who were community college transfers; the presence of articulation agreements between institutions that indicate comparable and transferable courses in mathematics and mathematics-related fields; actual task requirements, mathematical concepts, and skills developed in curricula of two year and primary receiving four-year institutions (see Chapter 1).*

**Advising/counseling available for choosing a major.** Sample indicators: *Student reports of having received good advice for choosing a mathematics-related major; faculty reports of the amount of time spent on advising pre-majors.*

**Transition from college to graduate school or mathematics-related jobs.** This transition would include the assessment of mathematics majors’ and mathematics-related majors’ post-college career plans (although one issue to be resolved is how these majors and careers might be accurately identified). This can be assessed through exit interviews of students in mathematics-related fields, or general senior surveys similar to the ones administered by the Higher Education Research Institute or other consortia of institutions.

### 4.3 Student interests, proficiency, and identity
This category of indicators is intended to capture students' cognitive beliefs, affective orientations, and mathematical proficiency. From one perspective, these indicators also reflect previous experiences students have had with mathematics courses, and can be divided into more distinct categories. Used in combination with demographic measures (gender, race/ethnicity, socioeconomic status), these indicators can be used to examine factors that may specifically impact particular groups. The work of Lantz (1985) and McLeod's (1994) review of research on affect in mathematics learning support a multi-dimensional representation of mathematics attitudes and have identified many of the indicators specified here as distinct constructs.

However, some researchers favor a parsimonious measure such as a single mathematics attitude index that incorporates items that tap into cognitive beliefs, affective orientations, and students' beliefs about their own proficiency (Miller, 1992). Furthermore, it is not clear whether these are attitudes regarding mathematics or whether the entire domain represents a students' mathematical identity. These questions, especially whether it is best to use a single index or multiple indices, is not intended to be resolved here. However, these become prime areas for future research and validation studies in the development of indicators because these constructs are key in determining students' decisions to participate in mathematics coursework or in mathematics-related careers.

There are also currently a plethora of mathematics attitudes scales that have been used in a variety of single-institution studies, as well as larger-scale studies. In particular, one of the most popular scales has been the Fennema-Sherman Mathematics Attitude Scales (Fennema & Sherman, 1976) developed in 1976 with a specific aim to understand gender differences in students' affective responses regarding mathematics experiences. The Mathematics Anxiety Rating Scale (MARS) has been more popular among those researchers interested in counseling psychology (McLeod, 1994). In addition, the Second International Mathematics Study (SIMS) contained several mathematics attitude scales to capture cross-national information regarding students' views of the role of mathematics in school, mathematics in society, mathematics self-
concept, and views of mathematics as a process (McKnight et al., 1987). Rather than advocate for the use of one particular measure here, and because most of these scales were developed for precollege student populations, we strongly recommend validation studies for the development and examination of these constructs among college students and subpopulations of interest (e.g. women, racial/ethnic groups, teachers in training, non-traditional college students). What follows are the general categories of indicators and sample indicators that would requirement development.

4.3.1 Cognitive beliefs about mathematics include:

Beliefs about the utility of mathematics in work and other settings. Sample indicators: Survey-based measure of the degree to which students believe that quantitative literacy is necessary to be successful in future endeavors; similar measured collected from faculty, parents, and peers.

Future educational goals expressed by students. Sample indicator: Percentage of students planning on obtaining degrees in fields which require advanced understanding of concepts related to mathematics.

Future career goals expressed by students. Sample indicator: Percentage of students at various educational stages planning on seeking employment in scientific, technical, or other mathematics-related fields.

4.3.2 Affective orientations toward mathematics include:

Preferences students express relative to additional exposure to mathematics (e.g., regardless of ability, past performance, and classroom experiences some students may simply prefer to study in other areas when given a choice). Sample indicators: Priority given to mathematics, relative to other areas of study, in a survey of student intentions; student enrollment in non-required courses which emphasize quantitative reasoning skills.
Confidence that students express in their ability to succeed in mathematics courses. Sample indicators: *Survey-based measure of the student confidence relative to different type and levels of mathematics, or self-reports of ability in mathematics relative to others of the same age-group.*

Anxiety that students feel toward mathematics. Sample indicators: *Student scores on a mathematics anxiety index; percentage of students withdrawing from mathematics courses reporting in exit interviews unmanageable anxiety.*

Enjoyment/Challenge that students experience. Sample indicators: *Degree to which students participate in co-curricular activities (such as mathematics clubs, or attending mathematics-related lectures); Percentage of students seeking independent-study in mathematics-related areas; Percentage of students who seek out additional, challenging problem sets; student ratings of particular courses in terms of the degree of enjoyment or intellectual challenge experienced.*

Mathematical identity indicators tap into students conception of themselves as mathematicians, or feelings about whether they are recognized for their mathematical skills. Sample indicators: *Percentage of students choosing to major or minor in mathematics, or in quantitative specialties within other disciplinary areas; Survey-based responses of the degree to which students view mathematics as being central to their personal self-perceptions, or the extent to which students view themselves in mathematically-oriented careers in the future; or the number of students winning institutional awards or recognition for mathematical skills or activities.*

4.3.3 Mathematical proficiency indicators determine the extent to which students decide to progress further at an institution in terms of course sequences and majors. Students have a tendency to elect courses and fields of study in which they feel some degree of competence. Institutional or faculty assessments of ability and performance, and how they are
communicated to students, play a large role in determining further student progress towards mathematics and mathematics-related courses and fields of study. Because how students’ proficiency is communicated is as important as assessing what students know (see Chapter 2), we suggest developing additional indicators regarding faculty encouragement to pursue further mathematics education at all levels of proficiency.

4.4 Current participation in mathematics education

These areas for the development of indicators reflect a broad definition of student participation in mathematics, acknowledging that quantitative reasoning skills can be developed inside and outside of the classroom. Perhaps more importantly, students’ experiences while participating in mathematics and mathematics-related courses may largely determine their decisions about whether to continue in course sequences, seek majors, and persist until degree completion in the field.

4.4.1 Course enrollment and type constitute a common category of monitoring the extent to which students participate in mathematics. Much of this information is monitored at the department level, but national course enrollment information is effectively monitored and shared by the Mathematical Association of American through their periodic surveys of undergraduate programs in mathematical and computer sciences. Such information may be lost, however, to policy makers who have come to depend on federal data for information about students, faculty, and institutions regarding mathematics and science participation. Furthermore, assessments of quantitative reasoning skills that occur in other departments are overlooked. Surveys of faculty that ask about major areas of teaching responsibility will determine the extent to which a variety of departments employ mathematics, statistics, and economics instructors to teach quantitative reasoning to undergraduates. In addition, surveys of a wider range of departments may monitor additional courses where mathematics learning occurs. In short, finding ways to cast a wider net for monitoring course enrollments may help identify the
extent to which colleges and universities are working to improve the quantitative literacy of undergraduates. This may help us identify the levels of mathematics preparation for students entering specific careers including, for example, the training of new teachers. In addition to surveys of departments and faculty in a broad range of departments, data regarding trends in course enrollments can be obtained through student transcripts. Sample indicators of course type/enrollment include: Actual proportion of student electing to enroll in non-required mathematics courses; the proportion of students electing different types of mathematics courses (e.g., pure versus applied).

4.4.2 Supplemental programs and parallel experiences that occur as part of the co-curriculum, or outside of classroom, have important implications for sustaining student participation in mathematics. These programs and experiences create peer environments for mathematics achievement, an important social and academic strategy for sustaining student interest and enhancing mathematical identity or improving self-confidence in mathematics. However, such programs are often enhanced by the special recognition from the community and/or opportunities to interact with others who have similar educational goals or proficiency levels. Monitoring student participation in such experiences that include internships in mathematics-related careers, participation in faculty research projects, mathematics clubs and team competitions, regular social activities with faculty, honors programs, workshops or special summer programs for enhancing student skills in a non-competitive environment. Surveys of students and institutions might reveal the extent to which supplemental programs and activities are made available to students and whether such experiences serve to encourage the development of mathematical talent among students.

4.4.3 Involvement, experiences, and classroom performance constitute an important influence on students and subsequent decisions to participate in mathematics. Researchers have consistently shown that the amount of engagement inside and outside
the classroom has a significant influence on a wide range of student social and academic outcomes (Astin, 1993; Kuh, 1991; Pace, 1984). Furthermore, student perceptions of the classroom climate have a definite impact on the extent to which students feel as if they “belong” or feel comfortable in a particular course or institution (Frazier-Kouassi, 1992). Sample indicators: *Student and faculty reports of engagement in classroom activity through group projects, senior projects, regular problem-solving activities; hours per week devoted to study or preparing for class in groups and individually; student perceptions of the classroom climate for interaction; student and faculty perceptions of the level of cooperation and competition in the classroom; student feelings of neglect or discrimination in the classroom.*

4.4.4 **Degree attainments** constitute the traditional pipeline indicators that we have successfully collected at the national level for many years now to monitor participation trends by gender, race/ethnicity, and institution type (National Science Foundation, 1992). To these data we recommend adding particular information that would help us understand more about mathematics participation. Specifically, combining information about time to degree for undergraduates in mathematics and mathematics-related fields and including additional information about how part-time, older students, linguistic minorities, foreign students, and new teachers fare regarding degree attainment.

4.5 **Changing Educational Practices**

While innovations in mathematics teaching are detailed in other parts of this volume, it is important to mention here that the effectiveness of such innovations must be judged in light of not only improved student performance outcomes but also increasing student participation in mathematics. This includes increasing the sheer numbers as well as the types of students participating in mathematics. Participation indicators detailed in the previous sections could be
used to evaluate changing educational practices such as the introduction of a new quantitative reasoning requirement, different teaching or pedagogical innovations, revisions of course sequences, or the development of new mathematics courses to meet the training needs of developing careers in technology. This involves:

- initiating comparative studies of the co-existing innovations and traditional educational practices, controlling for selection effects (e.g. comparing students’ subsequent participation relative to enrollment in mainstream and non-mainstream calculus).
- using institutional records of participation rates (degree attainments and course enrollments) by types of students to understand pre- and post-innovation effects on student participation, holding constant any changes in admissions standards that determine changes in the kinds of students pursuing undergraduate study.
- using attitudinal surveys to assess changes in students cognitive beliefs and affective orientations, in order to determine whether the innovation has markedly changed students attitudes towards mathematics.
- assessing the long range impact of specific interventions (e.g. summer mathematics camps, internships in mathematics-related careers) and curricular change through longitudinal surveys to assess student aspirations, behaviors and attitudes towards mathematics subsequent to participation in these distinct educational practices.

5.0 Developing indicators on student participation

In developing the indicators outlined in this chapter, a number of strategies should be considered. Existing data sets will be useful in developing a number of indicators, but many of these will likely be limited to providing pipeline-kinds of information. Moreover, many of the indicators described in this chapter deal with students experiences and perceptions, which cannot be culled from existing institutional records. Thus, in addition to short-term strategies, longer-term approaches will be considered along with related methodological considerations.
Whatever the source of data (new or existing) used to develop indicators, an important question which needs to be considered is the most appropriate level at which indicators should be developed. Table 1 shows a summary of potential indicators by category and level of data collection. Those indicators collected at the individual level can be aggregated to the institutional level, and indicators collected at the institutional level can also be reported at the national level.

As an example of how different approaches influence the kinds of indicators that can be developed, take the research designs employed by the NSOPF and the recent faculty surveys employed by the UCLA Higher Education Research Institute (Astin, Korn, & Dey, 1991; Dey, Ramirez, Korn, & Astin, 1993). The NSOPF employed a clustered sampling technique which was statistically efficient but, due to the small number faculty respondents per institution, makes the analysis of data at the level of the institution impractical. In contrast, the HERI approach sought responses from all faculty members at each participating institution, thereby allowing analyses at the institutional and, in theory, departmental level. This comparison is not put forth to argue that one approach should always be favored over the other, but that the issue of what questions might be answered using these data should not be considered separately from questions of methodology. Designing data collection schemes which can be used to answer questions at multiple levels may well lead to more complex designs and create practical problems needing careful coordination of cooperating agencies and institutions. Such systems also provide unmatched flexibility in providing indicators useful for addressing a wide variety of policy issues.

Table 1 shows a summary of potential indicators by category and level of data collection. Those indicators collected at the individual level can be aggregated to the institutional level. Indicators collected at the institutional level can also be reported at the national level, so long as there is some general agreement about the how the specific information is to be gathered and reported.
5.1 Short-term strategies

Given the existence of a good number data sets which are focused on different aspects of undergraduate education, it should be feasible to develop useful indicators in short-order at very low cost. The National Science Foundation reports on science and mathematics education indicators (National Science Foundation, 1994; in press) could serve as a useful model for developing and disseminating national level indicators focusing specifically on undergraduate mathematics education. For example, the most recent indicator report provides data from the NCES High School and Beyond Study which indicate that contrary to most notions of a closed pipeline, a good number of students who end up majoring in science and mathematics came to the field relatively late in their educational careers (Table 4-1). Constructing similar kinds of indicators exclusively for mathematics might be illuminating.

In addition to using existing national and (other large-scale, multi-institutional) data sets to develop indicators, data generated through normal institutional practices should be examined for their potential to generate useful indicators. Computerized records of student enrollments as recorded on transcripts could be used to develop insights into student course-taking patterns. Following earlier efforts to collect and analyze student transcripts (IRHE, 1994), a small group of institutions could work together in developing procedures for making this information available to interested parties in an easily accessible format. Ideally, this work would take advantage of existing standards for the electronic exchange of transcript information.

5.2 Longer-term strategies

In seeking to collect new data for use as indicators, a series of validation studies should be conducted. Many of the constructs highlighted here have not been systematically tested within the context of undergraduate mathematics experiences. In addition, it is fairly common to find in the research literature references to the same construct in several studies even though researchers are using different measures. Conducting validation studies and developing consistent measures of important constructs will be a necessary first step in developing a new framework of indicators
which will be useful in moving us beyond basic pipeline indicators. Beyond this basic consideration, it will be necessary to consider a broad array of methodological approaches. Since none of the approaches will be universally suitable or feasible for collecting all of the outlined indicators, it will be important to draw upon individuals with diverse perspectives and expertise in selecting the most appropriate approach to capture information associated with each indicator.

The most basic approach to collecting systematic information is through cross-sectional methodologies. The cross-sectional approach has been applied most often in developing traditional indicators based on an educational pipeline approach. A strength of this approach is that cross-sectional data collection readily lends itself to trend analysis when indicators are systematically collected over time, and thus is particularly well-suited for examining ways in which student participation in mathematics is changing. Examining the number of degrees awarded in mathematics-related fields over time using data collected through the U.S. Department of Education’s Integrated Postsecondary Education Data System (IPEDS), or trends among mathematics faculty identified by an analysis of responses from different cohorts of National Survey of Postsecondary Faculty (NSOPF) respondents, are examples of this type of approach.

In contrast, a longitudinal approach to collecting mathematics indicator data on individuals is much better suited toward understanding the relationships which may exist between system inputs, processes, and outputs. By collecting data from the same students, faculty, or institutions over time, it becomes possible to develop analytical models which identify how different educational processes or experiences are influencing mathematics-related indicators. Moreover, following the same cohort of individuals over time allows researchers and policymakers to gain a greater understanding of the choices students make at particular transition points in their undergraduate education. Data from the National Educational Longitudinal Study (NELS:88), the Longitudinal Study of American Youth (LSAY), and Cooperative Institutional Research Program’s (CIRP) follow-up surveys are examples of data collection activities of a
longitudinal design. Each of these data sets include a few, but not all, of the indicators described in this chapter.

Cross-sectional and longitudinal data collection approaches are not mutually exclusive. It is possible to combine these approaches to provide for both cross-sectional and longitudinal analyses. By maintaining an on-going series of cross-sectional data collection activities it is then possible, on any desired schedule, to longitudinally follow-up any number of cohorts who are represented in the cross-sectional data. The Monitoring the Future Project (focused on understanding patterns of drug use) and the CIRP are examples of on-going research effort employing this design.

Longitudinal data from two national surveys, High School and Beyond and NELS:88, also contain data on student transcripts. These data allow monitoring of mathematics course enrollment trends for a variety of students attending higher education for two generational cohorts. The data can provide indicator information on patterns of course enrollment, and proportion of students from various ability groups who elect specific types of mathematics courses, or trends in subpopulations enrolling in various levels of mathematics courses in college. This information may provide some indication of progress made in increasing mathematics enrollments, particularly for specific subpopulations of students. However, similar coursework among students across a variety of institutional contexts should not be taken to assume that students have had equivalent formal or informal educational experiences in mathematics (Eccles & Jacobs, 1986). Information derived directly from students and faculty through other methods are more useful in terms of determining the range of student responses to mathematics experiences encountered inside and outside of the classroom.

A final method of collecting indicator information is through the use of qualitative methodologies. A number of methodological specialties can be identified within a larger category of qualitative approaches (Patton, 1990), and could be used effectively as supplements or substitutes for more traditional quantitative approaches (especially in collecting information in
domains not readily reducible to survey or testing formats, such as institutional or classroom climate studies). Qualitative approaches would also be useful in developing and validating currently-untested quantitative indicators. Although usually associated with small-scale single-institution studies, an example of a multi-institutional qualitative study of science, mathematics, and engineering education is provided by Seymour and Hewitt (1994).

6.0 Summary

The changing nature of the student population in higher education and the implications that these trends have for student participation in undergraduate mathematics are important considerations in developing indicators. These trends mean that traditional metaphors for the mathematics education are less useful, and serve as an encouragement to focus on those factors which influence student intentions and their experiences within the educational system. In addition, continuing concerns about equity in access and participation make it important to collect indicator information for women, various racial/ethnic groups, and groups which are of interest due to policy concerns (such as adult students, teachers in training).

In order to deal with these concerns, a conceptual framework is presented to help organize categories of proposed indicators that influence student decisions to participate in undergraduate mathematics. These categories of indicators include the general social environment and institutional practices; student cognitive and affective orientations regarding mathematics; current formal and informal types of student involvement in the study of mathematics; and student participation in light of changing practices in undergraduate mathematics. A number of short and longer-term strategies are suggested in approaching the development of indicators to monitor undergraduate mathematics participation and develop information useful in the quest to increase the quantitative literacy of all undergraduates and promote the development of the most mathematically talented.
References


Figure 1
Organizing framework for mathematics indicators

Social environment & educational practices
- Social norms (general, parents, peers)
- Market value of careers associated with mathematical proficiency
- Changing educational practices
- Institutional requirements
- Faculty expectations

Decision to pursue additional mathematics education
- Occurs regularly, according to academic calendar
- Important transition points

Student interest and proficiency
- Cognitive beliefs
  - Beliefs about utility
  - Future educational goals
  - Future career goals
- Mathematical proficiency
- Affective orientations
  - Preferences
  - Confidence
  - Anxiety
  - Enjoyment/challenge
  - Mathematical identity

Current participation in mathematics education
- Course type / enrollment
  - Traditional math courses (including remediation)
  - Math applications (including statistics)
  - Math service courses
  - Quantitative reasoning courses
- Supplemental programs and parallel experiences
- Involvement, experiences, and classroom performance
- Degree attainment