Charting the Course:
Quality Indicators for Undergraduate Mathematics Education

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Charting the Course: 
Quality Indicators for Undergraduate Mathematics Education

Executive Summary

Faculty members, and especially department chairs, face many questions as they seek to assess the current status and direction of their undergraduate mathematics programs. The questions certainly include questions from these major categories:

- the mathematics department itself
- the curriculum (programs and courses) offered by the department
- the instructional staff -- those who deliver instruction to the undergraduates
- the classroom practices commonly found in the department, and
- the students--those who are recipients of the instruction and the clients of the program.

This report outlines what a department might do to structure its self study and to establish a set of questions whose answers could serve, over time, as a barometer of change in the various facets of the departments' activities. The resulting measures are commonly called indicators -- indicators of the quality present in the department's program.

The following questions help frame a set of indicators likely to be of use to department personnel faced with the challenge of gathering data and documenting the quality of their department's undergraduate program:

- **The Institution and Department**
  - What are our department's major goals?
  - What role does our undergraduate program play in these goals?
  - What commitment has our department made to the undergraduate program in terms of FTE and financial support?

- **The Curriculum**
  - What are our current requirements for undergraduate programs?
  - Have we made recent changes and are we contemplating future changes in the undergraduate program?
  - How does our curriculum relate to our students' goals?
  - How do we monitor our curriculum and who is responsible for it?

- **The Faculty**
  - What are our current faculty's specialties, academic qualifications and age distributions?
  - How do our faculty's interests relate to major facets of our undergraduate programs?
  - What faculty development activities have taken place in our department in the past three years?
  - What resources does our department make available to support and improve instruction?
Executive Summary, continued

- **The Classroom**
  - **Classroom instruction**
    - What teaching strategies are used in our classrooms?
    - What range of competencies is expected of students in the undergraduate program?
    - What kinds of technology do we make available to support and enhance classroom instruction?

- **Assessment of student learning**
  - What kinds of assessment activities do we use that go beyond the usual kinds of tests -- chapter tests, department final examinations, etc.?
  - Do we use assessment to promote learning rather than to simply assign grades to students and, if so, how?
  - Do we use assessment to provide important information about non-cognitive student outcomes (such as attitudes, opinions and beliefs) and, if so, how?
  - How do we use technology in assessing student learning and growth?

- **The Students**
  - Who are our students, where are they from, and where are they going?
  - What opportunities do we provide for our undergraduate students to take part in the scholarly and social life of the department?
  - What has happened to our graduates of 1-, 5-, and 10-years ago?
  - What views do our students, present and past, have of our department and its programs?

Questions such as these are often, if not typically, answered in the absence of reliable data. Decisions about program quality, student outcomes and staffing too often are based on anecdotal information, tradition, or conjecture. However, answers to these questions, and others like them, are essential information as a basis for molding a quality mathematics program for a department's students. A carefully designed and well developed set of indicators goes far in providing such direction.

This report provides a framework for collecting data systematically. This kind of careful collection of empirical data will enable a department to make more informed decisions for improving mathematics teaching and learning. The focus is on developing structured sets of related data that provide a basis for viewing both the current status and the likely direction of change in undergraduate mathematics programs. These indicators should provide guidance to department chairs, to evaluation committees at the university and state levels, and even to those who are interested in the status of undergraduate mathematics education at the national level.
I. Background

A. What are Education Indicators and Why are They Important?

An indicator is a numerical quantity that represents the condition or status of some entity or system. The use of indicators in various fields, such as public or economic policy, has been common for many years. (Burstein, Oakes and Guiton, 1992).

We are all familiar with using economic indicators to describe the health and direction of the nation's economy. These indicators -- for example, the rate of inflation, the cost of living index, and the gross national product -- provide benchmarks against which we can make judgments regarding where our economy stands and in which directions it is moving.

Such indicators serve at least three purposes:
- to **interpret** or **understand** relative performance across units, institutions or settings,
- to **monitor** what is taking place **over time**, and
- to help determine effects of **intervention** or **policy change** -- either across settings or over time.

Education indicators are used for the same purposes in examining educational systems. Mathematics instruction provided by a college or university is one particular educational system that can be characterized by indicators and shaped by well informed policies about educational practices.

**Definition:**

**Education indicators** are single or composite statistics that reflect important aspects of the education system (as economic indicators reflect aspects of the economy). They are expected to tell a great deal about the entire system by reporting the condition of particularly significant features of it. (Shavelson, et al. 1987, page 8.)

Consider, for example, the indicator data on student retentivity shown earlier. These data show enrollments in mathematics courses and programs from the first year of high school through completed doctorates. This indicator shows a dramatic drop-out
rate over successive years. The data portrayed prompted the following observation by the authors of National Research Council Report, *Everybody Counts* (1989):

> More than any other subject, mathematics filters students out of programs leading to scientific and professional careers ... Mathematics is the worst curricular villain in driving students to failure in school. (P. 7)

Developing a set of indicators for undergraduate mathematics requires careful thought and planning. No single indicator suffices, since single indicators cannot fully describe the complexity of any significant context. Instead, a web of related indicators is needed that allows one to triangulate on issues of central importance in describing any facet of interest in studying or planning to change a mathematics program. For example, different facets of providing collegiate mathematics instruction -- department, curriculum, faculty, classroom practices, and students -- suggest different statistical and descriptive vantage points for examining a mathematics program. The use of a single indicator to describe even one facet increases the likelihood of erroneous interpretations.

For instance, suppose one used drop-out rate compared to selectivity of admissions as an indicator without any other information concerning a program. Depending on how this indicator changes with time, one could make judgments about the relation of the two factors involved. But information is lacking to effectively guide the program through policies designed to bring about changes in this area. What program changes, for example, accompany changes in the selectivity rate? How does changing this rate change the composition of the student body? How do these changes affect the drop-out rates of different populations within the student body? Are there things that can be done to compensate for increased risks of dropping out for particular populations more at risk than others? Do the potential drop-out rates differ for different kinds of mathematics courses -for example, for calculus or for undergraduate statistics?

Looking only at one drop-out rate aggregate over all types of students and all types of courses, and looking at a similar aggregate rate of selectivity would likely hide as much as it revealed. These global, aggregate indicators are important -- as the earlier global indicator of retentivity demonstrated. However, for purposes of effective decision-making and planning, we are far better informed by knowing what affects
specific types of students and for which courses. These more specific indicators of drop-out rates and effects of selectivity allow us to target problem areas for special attention.

Other caveats about using indicators in describing undergraduate programs include concerns about the comparability of the settings to which indicators are applied. Use of an indicator -- by a mathematics department, another administrative unit, a state board of higher education, or a federal agency -- should take into account whether the facets and issues underlying a given indicator are comparable before using it in decision making or for comparative purposes. For example, viewing the undergraduate program's budget allocations in a small 4-year liberal arts college alongside a research university will necessarily be only partially valid because the two institutions have different missions, both in general and for mathematics instruction. However, the same budget allocation indicator could have considerable interpretive merit in comparing institutions with similar missions.

Indicators are best used to raise questions or identify potential problems. They are less useful for rendering summative judgments about the adequacy or performance of a program or institution. For this latter purpose they must be set in a context of related indicators and inform the common sense of experienced professionals. That is to say, when used as sources of information for reflection rather than for sole decision criteria, indicators can have a role in more insightful planning and policy making.

Consequently, indicators should be carefully constructed in structured sets of related indicators that provide full, rich, and contextually sufficient pictures of programs for both external and internal analysis. Externally, demands for indicators have gradually expanded from an exclusive concern with outcomes, to key questions of how instruction is organized and delivered. This expanded concern has been accompanied by calls for analyses of the "value added" by programs. Unfortunately, the requests for data on value added have often called for terse and truncated analyses, using an input-output model with only little attention paid to the qualitative differences by which programs add value to their "inputs" -- student types, instructional resources, etc. Internally, growing fiscal pressures and changes in numbers of students and what
they seek from mathematics instruction are leading many colleges and universities to take new looks at the mathematics instruction being delivered. That is, there is movement away from indicators that focus only on 'inspection at the end' of instructional delivery to those that focus more on continuously gathering data that describe how well 'core processes' are working.

B. Target Audience and Possibilities

This report has been written primarily for those with responsibility for developing and monitoring mathematics programs. At the campus level, this includes department chairs and individuals and committees who provide curricular leadership in mathematics departments. A more inclusive set of concerned individuals on a campus might include those in deans' offices and others responsible for the status and quality of curricula at the institutional level. The report also contains suggestions that might be useful for those in state boards of higher education offices or regional accrediting agencies.

Though secondary to this particular report, the questions explored here are significant for those at the federal level, as well. The questions raised provide guidance for developing and maintaining an indicator system to monitor the health of undergraduate mathematics education at the national level. The issues that are raised certainly try to move beyond "input-output" models of aggregate data to informed, process-sensitive data that reveal qualitative differences in how different outcomes are attained. Such indicators could provide valuable ideas for those concerned with undergraduate mathematics education initiatives at the National Science Foundation, private foundations, and those involved with undergraduate mathematics programs as parts of accrediting (pre-college) teacher education programs.

C. A Framework for Indicators for Undergraduate Mathematics

Establishing a set of indicators can provide baseline data as a foundation for monitoring the status, health, and direction of undergraduate mathematics in US 2- and 4-year colleges and in universities. Figure 1 presents a model that provides a
framework for thinking about how indicators describe the status, current directions, and quality of undergraduate programs in mathematics. Five clusters of issues related to undergraduate education in the mathematical sciences are identified. These major components are indicated in the vertical column on the model's left. These five areas (department, curriculum, faculty, classroom practices, and students) are explored further in the next chapter.

The placement of the topics in the column suggests a 'top down' view of undergraduate mathematics education. Mathematics education takes place in a variety of postsecondary institutions. Within these institutions, with different missions, departments have differing goals and priorities. It is the department that organizes the programs, develops a curriculum, selects faculty, staffs the courses, provides instructional opportunities for students, assesses student progress, conducts research, and otherwise provides much of the context within which the undergraduate mathematics program takes place. The department makes plans and creates the environment for the other aspects of mathematics instruction.

<table>
<thead>
<tr>
<th>Level</th>
<th>An Antecedents (including Intentions and Goals)</th>
<th>Tr Transactions</th>
<th>Ou Outcomes</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Department</td>
<td></td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>Curriculum</td>
<td></td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>Faculty</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IV</td>
<td>Classroom Practices</td>
<td></td>
<td></td>
</tr>
<tr>
<td>V</td>
<td>Students</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 1: An Organizational Framework for this Report

The three columns -- An, Tr, and Ou -- reflect foci that shape the processes that constitute a mathematics instructional program -- from institutional intentions to the

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1 Adapted from R. E. Stake.
evaluation of student outcomes. Column An focuses on the prior conditions and contexts for each of the five levels of the undergraduate program. For example, at the department level (Level I), the department's goals (Column An) develop from its mission and priorities. Column Tr focuses on departmental transactions by which these antecedent conditions are mediated in moving toward program outcomes. Column Ou examines the degree to which the departmental goals are attained (that is, the outcomes that flow from the antecedent conditions and the transactions).

Let us be clear on the kinds of things that would fall into the cells of the framework in Figure 1. Cell (I, An), for example, might contain indicators that reflect the program's current goals and intentions. It provides information on the recency of goals, the breadth of goals, and the consistency of goals and intentions. Cell (I, Tr) might include departmental actions to implement those goals. This could include the departmental structure for monitoring whether goals are attained and for initiating new plans to better attain or revise goals based on the attempt to implement activities to carry them out. Cell (I, Ou) would encompass indicators whose focus is on the attainment of departmental goals. For example, it might gather data on to what degree the various goals for the program are being met and by whom.

Similar descriptions can be provided for each of the other four areas (the rows, Levels II through V) of the undergraduate program -- that is, for the curriculum, the faculty, classroom practices, and the students. Section III of this report provides a proposed 'starter set' of indicators for the cells of Figure 1, using that figure as an organizing framework for identifying these illustrative indicators and their interrelationships.
II. Illustrative Quality Indicators

A. Institutions and Departments

Lower division collegiate mathematics is taught and learned in the United States in an extremely wide range of institutional settings, from community colleges to research universities. We provide students with greatly varying kinds of mathematical experiences across these institutions, their departments, and their programs of study. For example:

Size and complexity of the institution
Institutions providing mathematics instruction range greatly in size of student enrollment. Instructors' responsibilities vary from only occasional undergraduate contact to full-time assignments of teaching lower division courses.

Selectivity of admissions
Admissions policies vary among institutions. They range from highly selective admissions (for example, admitting only the top five percent of high school graduating seniors) to 'open admissions policies' under which all applicants possessing a high school diploma, or equivalent, may enroll.

Retention of students
Some institutions experience a high drop out rate of students from the institution, from particular mathematics courses, or from mathematics majors. Other institutions have comparatively stable student enrollments, with little movement among majors in a school year.

Articulation with 'feeder' high schools
In some institutions, especially community colleges, there often are close connections between high school and collegiate curricular offerings. The student population at such a college is typically predominately from the local 'catchment' area. In other cases, the student population is drawn from an entire state or region, or even from other nations.

(for further details see Ewell and Ray, Chapter 6)
Institutions and Departments

The question of a department's goals is central to assessing its health and vitality. What does it expect for and of students? What roles do departmental programs play in the institution's mission?

A. Some questions about the department:
   (QA1.1) What are our department's major goals?
   (QA1.2) What role does our undergraduate program play in these goals?
   (QA1.3) What commitment has our department made to the undergraduate program in terms of FTE and financial support?

(QA 1.1) What are our department’s major goals? (In particular, what is our department’s commitment to undergraduate instruction?)

Institutions, and departments within them, differ greatly in their orientation and missions. These differences have profound impacts on both departmental organization and resource allocation. In some institutions undergraduate mathematical sciences programs are divided among departments and faculties with majors in each of Departments of, say, Mathematics, Applied Mathematics, Statistics, and Mathematics Education. In other cases, these program areas are handled in one department. These differences influence what departments and faculties value and how they allocate their resources -- their financial resources, personnel, time, and interests.

Departmental missions may differ in the relative importance given to:
• how departmental and program goals relate to the institutional mission
• how the emphasis on undergraduate instruction compares to graduate teaching and research
• how emphases on mathematical applications and practice compare to those on theory and research
• how liberal arts and sciences compare to technical or career-oriented training programs.

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2 ‘Institution’ is used here as shorthand to avoid repeatedly writing “two and four year colleges and universities.”
**Sample Indicators: Undergraduate Teaching vs. Other Departmental Missions**

- Percentage of faculty FTEs devoted to undergraduate instruction compared to those devoted to service, graduate, or research efforts of the department. (IA1.1.1)

- Percentage of faculty reporting, respectively: undergraduate instruction, service course instruction, grant work, disciplinary research as their principal emphasis as a faculty member. (IA1.1.2)

- Percentage emphasis (weight) on undergraduate teaching included in department's promotion/tenure criteria and in annual resource allocation reports. (IA1.1.3)

(QA 1.2) What role does our undergraduate program play in departmental goals?

We need measures that indicate how far a department's policies and priorities actually reflect commitment to undergraduate instruction. These indicators are to describe ways in which undergraduate mathematics programs are high among a department's priorities, goals, and teaching efforts. One indicator of commitment is that of procedures in place for monitoring undergraduate programs and their quality. Does a department have a standing curricular committee, or subcommittee, devoted to its undergraduate program? If so, what is the role of this committee, or others, in monitoring the status and health of the undergraduate program? How and how often is an undergraduate program reviewed at the college, campus, or state level? Positive and thoughtful responses to these points can reveal different views of a program's status and of where that program is heading.

**Sample Indicators: Departmental priorities and policies concerning undergraduate instruction**

- Proportion of faculty reporting that they participate in committees/activities directly tied to undergraduate program goals and missions. (IA1.2.1)

- Are undergraduate program goals evaluated in reference to data on student career objectives and, if so, how often?

- How often are program goals discussed with those active in fields students have as career goals (e.g., engineering, business, etc.). (IA1.2.2)

(QA 1.3) What commitment has our department made to the undergraduate program in terms of FTE and financial support?

What resource allocations does the department, and the larger institution, make to support undergraduate mathematics programs? This would include allocating a portion of the department's FTEs directly for instruction or advising, as well as portions of the department's
Does the department have a regular budget for procuring and maintaining technological resources and, if so, what allocations in that budget are for resources that directly support undergraduate instruction? Has the department developed "smart" classrooms for the teaching of calculus and other central undergraduate mathematics courses? How do these expenditures compare to those for service courses, for graduate courses, for general faculty support, and for other department activities?

**Sample Indicator: Departmental Commitment to the Undergraduate Program**

| __ Proportion of tenured and tenure-track faculty involved directly in undergraduate instruction. (IA1.3.1) |
| ___ Ratio of undergraduate instruction credit hours generated by tenured and tenure-track faculty to credit hours generated by teaching assistants and adjunct instructors. (IAI.3.2) |
B. Curriculum

A mathematics department's curriculum consists of its programs, courses, and plans for delivering them. A department shapes its curriculum by its vision of the needs of the students it serves and how those needs are to be met so those students become appropriately competent in mathematics. What must be done to prepare participating students to enter their chosen careers and professions, and to meet both the informal as well as the formal, statutory requirements of the state or profession for mathematics preparation?

The curriculum also includes the department's vision of how students should learn and how they should be able to apply the knowledge -- the body of concepts, principles, and skills -- they acquire. In this regard, the following questions may be asked:

- Should students be given opportunities to learn primarily in group settings, in lectures, in project-oriented laboratory settings, or individually?
- What should students be expected to do to show they have mastered mathematical material -- complete multiple choice tests, write responses to open-ended questions and problems, construct models, develop action plans related to real-world problem settings, present talks discussing the material and its ramifications in a given problem situation, etc.?
- Who in the department is responsible for establishing curricula for the various undergraduate programs offered?
- What individual or group is responsible for monitoring how curricular plans are actually carried out in classes and other departmental instructional activities?

These questions all relate to curriculum design, delivery, and monitoring for an undergraduate mathematics program. Curriculum is more than simply a set of courses and examinations to assess the student growth in those courses. Ideally, curriculum is driven by the desire to attain goals as discussed earlier. Curricula evolve and change as they encounter the realities of the settings in which they are implemented. How does the department adapt to changes in partner disciplines that have mathematical requirements, to changes in student needs and student plans for their futures, and to changes in the mathematical sciences themselves--changes in mathematical content and in the goals and methods of delivering that content to students?

In this section we discuss how a department might establish indicators to inform the department about various aspects of its curriculum development and delivery processes. The box below contains questions to help move departments view curriculum more dynamically.
B. Some questions about curriculum (programs and courses)

(QB1.1) What are our current requirements for undergraduate programs?

(QB1.2) Have we made recent changes and are we contemplating future changes in the undergraduate program?

(QB1.3) How does our curriculum relate to our students' goals?

(QB1.4) How do we monitor our curriculum and who is responsible for it?

(QB1.1) What are our current requirements for undergraduate programs?

Undergraduate mathematics program requirements include sequences of required courses and also include the kinds and levels of student performances expected in these courses. A first broad brush stroke for examining a department's curriculum is mapping the courses that make up the "streams" in its undergraduate offerings. Course offerings include those for the institution's general education program, service courses (for those in pre-service teacher education, in some form of engineering, in business-related majors, etc.), courses for mathematics majors, and courses for mathematics literacy.

In order to better understand our own department's strengths and weaknesses, we may wish to know how these courses compare to offerings provided by peer institutions. We may also want to know to what extent do they meet recommendations from appropriate professional groups within mathematics (for example, the American Mathematical Association of Two-Year Colleges, Mathematical Association of America, American Mathematics Society, or National Council of Teachers of Mathematics), of professional career groups (for example, actuarial education, engineering accreditation, or the National Council for the Accreditation of Teacher Education), or of state boards or regents for higher education.

We may also ask, "What are mathematics students expected to do, both as a part of their formal study and afterwards?" Expectations vary greatly among departments and courses. Some curricula expect little of students -- perhaps only to learn basic facts, perform routine procedures and complete simple textbook exercises for a few problem types. Others require students to
complete internships and undergraduate research experiences where they are faced with original problem-solving situations requiring them to integrate their mathematical knowledge around concepts and generalizations drawn from disciplines and to employ mathematics in solving non-routine problems.
Sample Indicator: Expectations for student performance in mathematics

Data on the expectations of particular courses can be obtained by analyzing textbooks, course syllabi, assignments (both in class and for homework) and examinations.

Student performance expectations for departmental examinations (IB1.1.1)
- For each examination, for each class section, identify for each question and subquestion on the examination as many of the following performance expectations as apply. Report the proportion of responses for each course
  - Recall appropriate factual information and mathematical concepts relevant to a given situation.
  - Perform routine mathematical procedures (those that are primarily algorithmic and with limited contingent behaviors).
  - Understand or create an appropriate mathematical model (representation) of an everyday situation and express questions from that situation in terms of the model.
  - Provide plausible, mathematically-based justification for a problem solution strategy, conjecture, etc.
  - Read complex mathematical statements with understanding of the statement's logic, the meaning of symbols used, the syntax of mathematical expressions involved, and so on.
  - Construct a deductive proof of a well-formed conjecture.

The sample list of expectations above is not complete but rather only illustrative. Similar indicators could be calculated for homework assignments, course syllabi, or textbooks.

(QB1.2) Have we made recent changes and are we contemplating future changes in the undergraduate program?

Many changes and reforms have been recommended for curriculum and instruction in collegiate mathematical sciences programs over the past decade. At the two year college level, the American Mathematical Association of Two Year Colleges (AMATYC has produced guidelines for curriculum reform in its Crossroads in Mathematics report (AMATYC, 1995).

Recent calculus reform efforts suggested changes in the curriculum, in assessment practices, and in the mode in which instruction is delivered in the basic undergraduate calculus sequence. Other recommendations were made for changes in introductory probability and statistics sequences, shifting the curriculum to give greater emphasis to descriptive statistics, use of technology, statistical modeling, and exploratory data analysis. These changes were accompanied with calls for changes in class formats and for introducing projects into the basic statistics curriculum. More recently, recommendations have aimed at change in introductory

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3 Again, see also AMATYC's Crossroads in Mathematics report.
mathematics courses for both pre-service elementary and middle-school teachers. These recommendations have come from the NCTM Curriculum and Evaluation Standards, Professional Standards for the Teaching of Mathematics, and the MAA's Call for Change. Even more recent recommendations have expanded with new calls for reexamining the basic liberal arts introductory sequences to assure that they reflect a current view of the mathematical sciences -- one that includes aspects of discrete mathematics, decision sciences, as well as glimpses of the core areas of algebra, geometry, probability, and statistics.

<table>
<thead>
<tr>
<th>Sample Indicator: Changes in curricular delivery</th>
</tr>
</thead>
<tbody>
<tr>
<td>Indicate with a √ which of the following emphases and/or activities are present in the department's courses and programs (QB1.2.1):</td>
</tr>
<tr>
<td>__ use of writing (English sentences and paragraphs rather than only mathematical expressions)</td>
</tr>
<tr>
<td>__ emphasis on modeling</td>
</tr>
<tr>
<td>__ emphasis on multiple representations of concepts (graphical, symbolic, tabular, etc.)</td>
</tr>
<tr>
<td>__ use of small groups</td>
</tr>
<tr>
<td>__ use of extended projects</td>
</tr>
<tr>
<td>__ use of alternative modes of assessing student learning (portfolios of work, projects, written reports, extended open-ended problems)</td>
</tr>
</tbody>
</table>

(QB2.3) How does our curriculum relate to our students' goals?

Here we are concerned with the extent to which the curriculum that we offer relates to student goals. Questions to consider include: What percent of students taking departmental courses are also mathematics majors? What percent are taking courses required in other majors? What percent are taking courses simply to fulfill distribution requirements of the institution's general education program? What percent are taking courses out of personal choice rather than to fulfill a requirement? Which courses are those taken by these groups of students? How satisfied are with the department's array of offered courses? How do they feel about the frequency and rotation of these course offerings?

It is also important to know where departmental program graduates believe that the courses offered by the department met their needs as they moved from undergraduate education to career or to graduate school. For example, do our graduates believe that the courses covered the appropriate topics? Do they feel the level of expectations was appropriate to prepare them for the needs of their new positions? Did the use of technology and communication skills departmental courses match the needs they have found in their new positions? Did their mathematics preparation allow them to pursue the careers or graduate studies of their choice?

<table>
<thead>
<tr>
<th>Sample Indicator: Summary of partner disciplines representatives (e.g. department chairs) ratings of the mathematics department's calculus sequence (IB.1.3.1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Partner discipline (illustrative only)</td>
</tr>
<tr>
<td>----------------------------------------</td>
</tr>
<tr>
<td>Engineering</td>
</tr>
<tr>
<td>Natural Sciences</td>
</tr>
</tbody>
</table>
How do we monitor our curriculum and who is responsible for it?

It is important to monitor the curriculum from various perspectives. For example, we need to take into account what partner disciplines (e.g., engineering, business, computer science) need from mathematics coursework for their students. These needs change with the evolution of those disciplines, with the changing nature of mathematics, and with the impact of changes in mathematics and its uses on those disciplines. This makes it necessary to periodically review the content and organization of mathematics programs and courses. Some courses no longer fill the traditional needs that shaped them; some are counted on to fill newly emerging needs. Thus courses and their content must be periodically brought in line with those who most use them.

Mathematics departments differ in how regularly and consistently they review their curriculum. They also differ in how others outside of the department (especially in the partner disciplines) are involved in the review process and in the mechanisms available for initiating and carrying out course reviews and revisions. They differ in their support for developing new courses and in the kind of information available (for example, ratings of student satisfaction and knowledge gained) as course review and revision is carried out.

Mathematics continues to be viewed as the 'queen and servant' of the sciences. This role is reflected in mathematics departments' relationships to 'partner disciplines' departments—disciplines which have grown to include the life sciences and economics and finance. Mathematics curricula (programs and courses) must satisfy not only mathematics department programs needs, but also programmatic needs for partner disciplines' majors.
Sample Indicator: Checklist on periodic and sustained review of the curriculum. (IBI.4.1)

Mark with a √ whether the department provides for:

- regular review of its academic programs in terms of its own majors
- regular review of its academic programs in terms of the needs of partner disciplines
- ‘partner discipline’ participation in setting curricular priorities for service instruction
- regular revision of existing courses
  - by departmental initiative
  - by individual faculty initiative
- proposing new courses
  - by departmental initiative
  - by individual faculty initiative
- determining, from a longitudinal perspective, aspects of student growth and development such as:
  - enrollment and performance in subsequent courses
  - career paths (especially mathematics-based careers)
  - attitudes toward mathematics as a discipline
C. Faculty (instructional staff)

The department's instructional staff, especially those involved in the undergraduate program, serve as a rich source of indicators to describe the current status and potential future direction of the program. It is the instructional staff that implements, or brings into being, the department's goals and objectives.

Here are some key questions to consider as one decides what data to gather to provide indicators of the nature of the staff that has responsibility for delivering instruction.

<table>
<thead>
<tr>
<th>Some questions about the faculty (instructional staff)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(QC1) What are our current faculty's specialties, academic qualifications and age distributions?</td>
</tr>
<tr>
<td>(QC2) How do our faculty's interests relate to major facets of our undergraduate programs?</td>
</tr>
<tr>
<td>(QC3) What faculty development activities have taken place in our department in the past three years?</td>
</tr>
<tr>
<td>(QC4) What resources does our department make available to support and improve instruction?</td>
</tr>
</tbody>
</table>

(QC1) What are our current faculty's specialties, academic qualifications and age distributions?

Selected characteristics of the instructional staff provide essential background information needed for long range planning - for program development, instructional support, and future hiring needs. Information on staff interests includes their areas of expertise; academic background, years in rank, gender, ethnicity, and age.

Sample Indicator: Instructional staff: Specialties, academic background and age distributions (IC1.1)

- Number and proportion of faculty in each disciplinary sub-area, (e.g., algebra, analysis, statistics) (IC1.1.1)
- Number and proportion of faculty in each 5 year age range, by level of academic preparation. (IC 1.1.2)
- Proportion of faculty in each 5 year age range broken out by disciplinary specialties, gender, rank, expertise with technology. (IC1.3)
- Planned recruitment of faculty by discipline specialty for the next five year period of time (numbers and specialties) (IC1.1 A)
- Results of last recruitment efforts - number of candidates and record of offers versus acceptances. Data on salary competitiveness and rank competitiveness. (IC1.1.5)
(QC2) **How do our faculty's interests relate to major facets of our undergraduate programs?**

The interests of a department's instructional staff in various aspects of its undergraduate program provide key starting points for planning staff and program development activities. How involved are faculty in the design and monitoring of the undergraduate program is one such indicator. Another indicator is the proportion of faculty actively engaged in activities for undergraduates such as sponsoring mathematics clubs, coaching those preparing for special examinations or contests, or counseling and program advisement.

### Sample Indicator: Faculty interest/involvement in the undergraduate program

- Proportion of the faculty involved in significant extra-class activities associated with program design and monitoring (IC2.1)
- Proportion of the faculty involved in significant extra-class activities associated with student activities such as math club, Putnam team, actuarial exam preparation (IC2.2)
- Proportion of the faculty involved with undergraduates in significant program or other advisement/counseling activities (IC2.3)
- Proportion of the faculty involved in extra-class student activities (IC2.4)

(QC3) **What faculty development activities have taken place in our department in the past three years?**

A departmental or institutional support network that promotes continuing professional development of the instructional staff enhances the vitality of a department's instructional program. This suggests that a critical question is to what extent there are available in the department resources for the staff to maintain and upgrade their expertise in course planning and instructional delivery skills - for example, in classroom uses of technology (graphing calculators, the Internet, etc.)?
Sample Indicator: Faculty development activities in the past three years (IC3)

To what extent were the following available (mark all that apply):

- Workshops on single topics (example: Using graphing calculators)
- Multiple session, in-depth ‘courses’ on effective instruction
- Orientation sessions for all new instructional staff on effective teaching
- Arrangements in the department for mentoring concerning teaching performance
- Provision for observation of classroom teaching by peers or by outside specialists
- Periodic seminars or discussion groups on current trends and issues in the teaching and learning of undergraduate mathematics
- Formal training in teaching provided to teaching assistants as a part of their academic program
- Physical support (office space, telephone access, e-mail, etc.) provided to part time faculty and TAs to help them discharge their instructional role.

(QC4) What resources does our department make available to support and improve instruction?

The extent and nature of instructional resources available to support undergraduate instruction help define the overall quality of the program. This support includes, among other things:

- Resources for assisting instructional staff to develop their classroom teaching proficiency
- Support for a diverse menu of instructional activities that ranges from disciplinary content to pedagogical approaches.

Sample indicator: Extent and nature of available instructional resources

- **Extent of departmental resources for professional improvement** (IC4.1): formal faculty development programs; seminars; colloquia; visiting speakers, etc.

- **Levels of utilization** of instructional improvement opportunities, by type of instructional staff member. (IC4.2)

- **Nature of coverage** of instructional improvement activities in terms of such issues as the relative attention dedicated to disciplinary content vs. approaches to pedagogy (IC4.3)

- **Length of time** that members of the department have been engaged in instructional improvement activities. (IC4.4)

(for further details see Ewell and Ray, Chapter 6.)
D. Classroom practices and assessment

Curriculum is implemented by delivering instruction and opportunities for students to learn. For each course, and each section of that course, transactions take place among instructor and students that help to attain the goals for that particular course.

Questions about classroom instruction

(QD1.1) What teaching strategies are used in our classrooms?
(QDI.2) What range of competencies is expected of students in our undergraduate programs?
(QD1.3) What kinds of technology do we make available to support and enhance classroom instruction?

Indicators may be developed that center upon detecting and monitoring 'good practice' as a proxy for instructional quality. The characteristics of 'good practice,' however, are likely to be the center of necessary departmental discussions in developing an indicator system to inform their self-understanding. Opinions on the nature of 'good practice' will likely differ. Recent reforms have suggested, essentially, that any candidate for 'good practice' be contrasted with a 'traditional' teaching model that considers students as passive recipients of delivered knowledge. In contrast, active learning techniques stress student discussion and group work. Another theme is frequent feedback on performance - practice based on the belief that students learn better when given prompt responses about their own performance on tests or problems. (Ewell, chapter 6)

(QD1.1) What teaching strategies are used in our classrooms?

Active learning

Many leaders in undergraduate mathematics curriculum reform place a premium on changes in pedagogical style. They emphasize that students should be active learners and should learn to think autonomously. To help this happen, many believe that students should work together in small groups, because they feel small group work tends to replace competition with cooperation, to promote conversations about mathematics, and to provide opportunities for students of different strengths and learning styles to contribute to solving problems. They argue, moreover, that students should acquire the habit and skill of working in teams because that will often be expected in their professional life. (Schoenfeld and Dossey, Chapter Two, pages 9-10)
Consider, for example, the following excerpt from the report of a site visit to a college campus:

'The classroom atmosphere created by the faculty allows students to freely ask questions and voice their opinions. In one calculus class that was visited, the students were actively involved during the entire period. Informal groupings of students helped each other while the teacher presented problems and helped groups when they called upon her. These classroom experiences carry over into informal study sessions in the mathematics lab and dormitories ... It appears that the day-to-day activities that make the Southern University mathematics program work for its students are not that different from those at other schools visited. The faculty is always seeing ways to improve its courses and programs'.


Some of the aspects of the effective mathematics programs that were identified by this site visit can be captured by indicators such as those described below.

### Sample indicator: Active participation in class work

(Data for this indicator could be obtained from a survey of mathematics department administrators.)

- Proportion of class sections using small group activities at least once per week (ID1.1.1)
- Proportion of students regularly (at least once in a class period) interacting with each other about mathematics
  - in class generally (ID1.1.2)
  - when given the opportunity to do so in class (ID1.1.3)
  - in laboratory or discussion sections (ID1.1.4)

### Multiple representation of concepts

Mathematics instruction has often represented its content symbolically and verbally. Recent reform efforts have suggested that instruction use multiple representations of concepts, a departure from the verbal development that characterizes many 'standard' lectures. Proponents of this approach feel that students gain power and clarity of understanding by becoming familiar with varieties of concept representations -- from graphical to tabular to verbal to symbolic -- and by developing the ability to translate among these representations. (Schoenfeld and Dossey, Chapter Two, page 53, adapted.)
Sample indicator: Use of multiple representations of concepts (ID 1.1.5)

Proportion of classroom activities in which concepts are developed that involve the use of these representations:

- graphic
- symbolic
- verbal
- numeric (tabular)

(QDI.2) What range of competencies is expected of students in our undergraduate programs?

Recent developments in curriculum (such as the NSF calculus reform initiatives) have expanded the scope of what is considered as mathematical 'content'. Broadly speaking, an indicator system should attempt to capture both the traditional content knowledge base and related discipline specific processes. (Schoenfeld and Dossey, Chapuz Two, page 11)

Sample Indicator: Competencies that are expected of students in the undergraduate program (IDI.2.1)

For each mathematics course, identify the following features of its knowledge base:

- Major facts, concepts and principles the students are expected to learn;
- Major procedures and techniques the students are expected to know, and the kinds of computations (with and without technology) they should be able to perform;
- Knowledge about mathematics: its nature and history
- Kinds of reasoning and sense-making in which the students are expected to engage (e.g. quantitative, spatial, symbolic, relational, probabilistic, logical.)
- Kinds of representations that students are expected to be able to employ (e.g. graphs, tables, matrices, sketches)
- Kinds of connections, within and outside of mathematics, that students are expected to be able to make;
- Ability to communicate (e.g. read, write, speak, listen and model) mathematically.
(QD1.3) **What kinds of technology do we make available to support and enhance classroom instruction?**

Today a wide array of educational technology is available for classroom instruction. Technology has the potential to enhance instruction and to provide students with critically important workplace competencies by helping them become proficient in a variety of mathematics-oriented technologies.

---

**Sample Indicator: Technology to support and enhance classroom instruction** (ID1.3)

For each undergraduate course, indicate which of the following technologies and classroom uses are employed at least once per week.

<table>
<thead>
<tr>
<th>Technology</th>
<th>Classroom Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>__ Calculators</td>
<td>low level numerical methods, graphing</td>
</tr>
<tr>
<td>__ Calculator Based Laboratories (CBLs)</td>
<td>exploring slopes, first and higher derivatives</td>
</tr>
<tr>
<td>__ Micro-computer Based Laboratories (MBLs)</td>
<td>curve-fitting</td>
</tr>
<tr>
<td>(distance sensors, timers, etc.)</td>
<td></td>
</tr>
<tr>
<td>__ Computational packages</td>
<td>linear programming, differential equations</td>
</tr>
<tr>
<td>(including statistical software)</td>
<td>regression analysis, analysis of variance</td>
</tr>
<tr>
<td>__ Symbolic computation software</td>
<td>solving equations graphically and</td>
</tr>
<tr>
<td>(computer algebra systems: Mathematica, DERIVE,</td>
<td>symbolically</td>
</tr>
<tr>
<td>Maple, etc.)</td>
<td></td>
</tr>
<tr>
<td>__ Electronic networking</td>
<td>downloading software and datasets, and</td>
</tr>
<tr>
<td>high end computing (e.g. Java)</td>
<td>real time dynamic modeling</td>
</tr>
<tr>
<td>__ Video-based laboratories</td>
<td>Modeling: trajectories, orbits, etc.</td>
</tr>
</tbody>
</table>
D. Classroom (continued)

Assessing student learning

The one, overarching question to be asked at every stage of a student's mathematical education is: "What mathematics do students know and what are they able to do with that mathematics?" As curriculum goals have expanded, students are being asked to engage in a much wider variety of mathematical tasks, and to make use of a greater range of mathematical processes.

Some questions about assessing student learning

(QD2.1) What kinds of assessment activities do we use that go beyond the usual kinds of tests -- chapter tests, department final examinations, etc.?

(QD2.2) Do we use assessment to promote learning rather than to simply assign grades to students and, if so, how?

(QD2.3) Do we use assessment to provide important information about non-cognitive student outcomes (such as attitudes, opinions and beliefs) and, if so, how?

(QD2.4) How do we use technology in assessing student learning and growth?

(Q3b1) What kinds of assessment activities do we use that go beyond the usual kinds of tests -- chapter tests, department final examinations, etc.?

Current methods of gathering information about student performance are likely to be inadequate for some aspects of student performance, especially some of the more recent suggestions of complex and integrated student activities. It is unlikely, for example, that the ability to communicate mathematical ideas can be adequately captured by multiple choice tests. Similarly, using extended projects, especially group projects, calls for re-thinking assessment practices.
The item below, for example, requires a broad range of capacities on the part of the student.

(For further information, see Schoenfeld and Dossey, Chapter 4)

**Sample Indicator: Assessing a wide range of student capacities: Interpreting graphs (ID2.1)**

Match the stories with three of the following graphs and write a story for the remaining graph.

(a) I had just left home when I realized I had forgotten my books so I went back to pick them up.
(b) Things went fine until I had a flat tire.
(c) I started out calmly, but sped up when I realized I was going to be late.

![Graphs](image-url)

Figure 1. Which story goes with which graph?

(QD2.2) **Do we use assessment to promote learning rather than to simply assign grades to (that is, sort) students? If so, how?**

Since 'mathematical performance' is multi-faceted, it must be examined in various ways.

- performance tied to basic facts, concepts, skills, and procedures
- performance tied to particular course goals
  - For example, such goals differ substantially for different calculus courses
- assessing broad mathematical understandings
  - For example, problem solving or critical thinking
- student beliefs and attitudes
  - For example: to become more confident in the use of mathematics and to value mathematics as a positive force in one's life.

The sample item below, though complex to score, can provide an abundance of information on different facets of mathematical performance.

**Sample Indicator: A freshman calculus project** (ID2.2.1)

**The Seattle Kingdome**

Your problem solving team has been hired as consultants to find the volume of the dome part of the Kingdome and then total volume of the Kingdome.

The dimensions are given as (sketch is not to scale):

a. The largest distance across is 660 feet.
b. The height from the floor to dome top is 250 feet.
c. The height of the vertical walls to where it starts to curve is 130 feet.

For the volume of the dome portion, in addition to the volume you compute, include:

- A sketch showing a representative piece.
- A representation of the i-th term of your Riemann sum.
- A description of what an individual term of the sum represents.
- A general Riemann sum to approximate this volume.
- An integral to represent the volume of the dome.
Sample Indicator: A freshman calculus project (ID2.2.1) continued

The Kingdome has a hydronic heating system which includes three boilers that produce together 16,500,000 BTUs per hour. It takes 0.0345 BTUs per cubic foot to raise the temperature one degree.

On Saturday, February 17th at 11:00 am the Kingdome doors will open for the annual Seattle Home Show. The second part of your task as a consultant is to determine when the maintenance crew should turn this heating system on in order to bring the temperature from a predicted forty-five degrees to seventy-one degrees (the standard temperature for such an event). The Kingdome must be up to temperature in time for the 11 am opening on the seventeenth.

Problem courtesy of Betty Hawkins, Shoreline Community College, Seattle Washington.

(QD2.3) Do we use assessment to provide important information about non-cognitive student outcomes (such as attitudes, opinions and beliefs) and, if so, how?

Non-standard assessment methods, such as using portfolios (collections) of student work, can help provide key information about students' accomplishments that are not easily available through traditional testing or homework and about students' dispositions toward mathematics. In contrast to most testing situations, which tend to document what a student cannot do, portfolios allow students to document what they can do. This medium enables students to demonstrate the learning and understanding of ideas beyond the knowledge of facts and algorithms.

Sample Indicator: Portfolios of student work (ID2.3.1)

What goes into a portfolio should, obviously, depend on the instructional goals of each situation. Typically, a portfolio includes a spectrum of student work -- some of which is optional (e.g., "Choose two pieces that you think exemplify your work at its best, and explain why you think they do.") and some of which is mandatory (e.g., students in a particular course must include one laboratory report, one open-ended exploratory problem, and one report of an extended collaborative project). Typically students are asked to include in the portfolio a "cover letter" for reviewers, which explains why they have chosen the entries they have, and what the reviewer should look for in them. Writing such a letter can be a powerful occasion for student reflection....

(Reference: Schoenfeld, A. ed. Student assessment in calculus. p. 19)
(QD2.4) **How do we use technology in assessing student learning and growth?**

Three decades ago, the role of technology in mathematics education was largely restricted to mainframe computers and special courses devoted to their use in engineering, the sciences and other domains with relationships to mathematics. In rapid succession technological change has transformed computational power from room-sized computers to programmable calculators and, most recently, to complex graphical capacities in personal computers and only marginally less sophisticated graphical capacities in hand-held programmable scientific calculators. Each change has introduced new possibilities for classroom demonstration, for individual student use of powerful tools, and new demands on assessment practices. The sample assessment item below illustrates a classroom application of the graphic calculator.

<table>
<thead>
<tr>
<th>Sample indicator: Using technology to assess student learning (ID2.4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>The conductivity of a certain alloy of steel can be modeled by the equation $-6e^{(2-x)}(x-2)^2 + 4$ where $x$ is the temperature and $y$, the thermal conductivity. Another substance is added to the alloy, and the conductivity of the resulting new alloy is found to be modeled adequately by: $\ln(x^{1/2}) + 1$</td>
</tr>
<tr>
<td>At what temperatures is the conductivity the same for both the original and the new alloy? Solve graphically.</td>
</tr>
</tbody>
</table>

Item continued on next page....
E. Students

The Mathematics Pipeline

The metaphor of the mathematics pipeline is useful to help portray in a dramatic way some of the problems facing undergraduate mathematics education in promoting and maintaining an adequate pool of candidates prepared and likely to pursue advanced degrees. The inadequacies of the pool for minorities and women can be especially documented.

However, limitations of the pipeline metaphor include:

- its inability to suggest alternative strategies for increasing student participation in mathematics,
- its applicability primarily to students enrolled in major fields of study offered through mathematics departments -- reinforcing the notion that learning mathematics is confined to a limited number of academic fields, and
- its appropriateness only for following the progress of traditional college-age students (not, for example, re-entry students), even though, for increasing numbers of students, there are multiple points of departure from and entry to the field of higher education.

A more appropriate image may be that of a metropolitan mass transit system, characterized by:

- many different points of entry and departure,
- multiple destinations (which may change several times en route!) and
- different routes (and modes of transport) available to reach the destination.

Furthermore, the transit metaphor captures aspects of students' experiences. For example: students may experience the same class in different ways, just as the experience of a bus ride is affected by such factors as:

- the skill of the driver,
- the condition of the bus, and
- the number and nature of the other passengers.

(Hurtado and Dey, Ch 5)
Some questions about the students:

(QE1) Who are our students, where are they from, and where are they going?

(QE2) What opportunities do we provide for our undergraduate students to take part in the scholarly and social life of the department?

(QE3) What has happened to our graduates of 1-, 5-, and 10-years ago?

(QE4) What views do our students, present and past, have of our department and its programs?

(QE1) Who are our students, where are they from, and where are they going?

There have been major changes in the aggregate composition of American college students along a variety of dimensions over the past several decades. Demographically, there have been sharp increases in the number of older students, women, and racial/ethnic minorities represented at the undergraduate level. The historical pattern of gender under-representation within higher education has been largely reversed so that women now represent the majority of students in higher education in general, and constitute as well the majority of entering first-time, full-time freshmen.

Enrollment patterns have been changing in other ways. Adults over the age of 25 have been the fastest growing group of college students, currently representing over 40 percent of all students in higher education. Since 1965, there has also been a shift toward increased part-time enrollment in higher education, with part-time students now representing about 43 percent of all students.

(for further information, see Hurtado and Dey, Chap 5)
Sample indicator: Background data on students enrolled in mathematics courses

Student demographics (IE1.1)
- percentage of students that are black or Hispanic
- percentage of residing within 20 miles of campus

Student enrollment behavior (IE1.2)
- percentage of students attending part-time
- percentage of first-time, full-time students completing programs
- average time to degree completion

Student preparation levels (IE1.3)
- percentage of new first-time students assessed as requiring remediation in mathematics on entry
- percentage of new first-time students achieving advanced placement in mathematics

Student enrollment modes (IE1.4)
- percentage of enrolled students completing course requirements through distance-delivery education

(QE2) What opportunities do we provide for our undergraduate students to take part in the scholarly and social life of the department?

Given the changing composition of student cohorts and their involvement with full-time instruction, new efforts are needed to help students have an opportunity to take part in a department's scholarly and social life. Frequent opportunities for out-of-class contact between faculty and students appear both important but more difficult. Student-faculty conversation is probably the single most often-cited feature of effective academic environments and, in most studies, have exhibited strong, independent effects on student development. An important qualification on this finding, however, is that such contact must not be purely social, but should involve an element of learning as well. The degree to which a given departmental (or institutional) environment values and encourages such contact may therefore be especially promising as an indicator of the quality of the undergraduate mathematics education it provides. It may also be useful to monitor the extent and quality of such non-face-to-face media for contact as E-mail networks and interactive Internet-based video.

(For further details, see Ewell, Chapter Six.)
Sample Indicator: Active 'student centered' policies, values and behaviors

- Emphasis placed on contact with students outside of class by individual faculty members on a survey or in interviews (IE2.1)
- Observed frequency of contact between faculty and student immediately before or after class (IE2.2)
- Presence of opportunities for faculty/student contact in the form of study groups, informal study sessions, mathematics clubs, etc. (IE2.3)
- Overall student feelings about faculty contact and accessibility as reported by survey or interviews. (IE2.4)

(QE3) What has happened to our graduates of 1-, 5-, and 10-years ago?

A changing student body puts increased importance on data about the career tracks of the department's graduates, data that can provide important information to take into account as the programs are reviewed and revised. The proportion of placements, the kind of placements and advancement reported in those placements are useful data for program evaluation purposes.

Sample Indicator: Follow up data on our graduates of 1, 5, and 10 years ago. By number of years beyond graduation, and by academic degree.

- Proportion employed in mathematics or science-based careers (IE3.1)
- Proportion providing leadership in mathematics or science-based careers (e.g., executive position in industry or research) (IE3.2)
- Proportion pursuing advanced degrees in mathematics or science-based fields (IE3.3)

(QE4) What views do our students, present and past, have of our department and its programs?

Departments may have in place visibly established mechanisms for listening to students -- determining their concerns, problems, and needs. Many such mechanisms will be informal -- arising through direct contacts between faculty and students. Others, though, are more explicit -- for example, needs assessment questionnaires, student satisfaction surveys, or "suggestion boxes." Active student associations directed toward bringing to the surface and discussing common student concerns may also be helpful in providing evidence of a commitment to take seriously student perceptions as data about the quality and effectiveness of departmental undergraduate instruction.

(For further information, see Ewell, Chapter Six.)
**Sample Indicator: Policies and procedures that treat students as "partners" or "valued customers."**

___ Extent to which student reports are positive concerning the attitudes of staff who are in direct contact with them about solving a particular problem or obtaining basic information (IE4.1.1)

___ Degree of readability and general "user-friendliness" of departmental (and institutional) publications describing basic academic policies and procedures (IE4.1.2)

___ Reported incidence by faculty of helping students solve individual problems (IE4.1.3)

**Sample Indicator: Identifying student needs and difficulties**

___ Presence of, and use of results derived from, student satisfaction surveys or reaction questionnaires (IE4.2.1)

___ Extent to which student responses on surveys (or in interviews) indicate feeling that their needs and opinions are listened to and taken into account by departmental personnel (IE4.2.2)

___ Extent to which data (provided, for example, by surveys and interviews) from graduates indicate that the department’s curriculum is achieving its goals in terms of student learnings and student habits of mind and work in the mathematical Sciences (IE4.2.3)
III A "Starter Set" of Indicators

In the Overview, we discussed a model for constructing indicators for the nature and quality of an undergraduate mathematics program. This model was represented in as a matrix. This matrix has 15 cells reflecting the interaction between, on the one hand, the program features of antecedents, transactions, and outcomes and, on the other hand, the institutional levels of department, curriculum, faculty, classroom practices, and students. The model appeared as shown below. The cells, when filled with appropriate indicators allows for the development of reports, predictions, and assessments of current status of a given institutions programs. It may also provide a basis for a federal set of indicators of the health and trends of mathematics education at the national level.

<table>
<thead>
<tr>
<th>Program level</th>
<th><strong>Au</strong> Antecedents (e.g. Intentions/Goals)</th>
<th><strong>Tr</strong> Transactions</th>
<th><strong>Ou</strong> Outcomes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Institutional level</td>
<td>I  Department</td>
<td>II  Curriculum</td>
<td>III  Faculty</td>
</tr>
<tr>
<td></td>
<td>IV  Classroom Practices</td>
<td></td>
<td>V   Students</td>
</tr>
</tbody>
</table>

Figure 2: An Organizational Framework for Indicators

Developing indicators requires considerable effort. Much of that work is idiosyncratic and particular to the institution or department seeking to examine and monitor the quality of undergraduate mathematics education. This brief report has served mainly as a sampler to show something of the possibilities of a thoughtfully developed indicator system. Some, however, may desire something a little more prescriptive in starting their thought on indicators. The present section addresses that concern.
The following is a proposed set of 15 indicators that the authors believe would provide a department, or even a federal agency, with a good start at describing programs of undergraduate mathematics education in a way that both helps evaluate program quality and effectiveness and could also provide insight into needed program changes. These suggested indicators comprise only a minimal set of information. They would need to be supplemented and triangulated with other points of reference and data. The weaving of such a cross-categorized web of information sources would corroborate findings. It would also suggest new connections that could both strengthen decision-making and interpretations for future directions, and facilitate identifying needed additional program support.

**Level I: Departmental Indicators**

**An: Antecedents**

**Sample Indicators: Undergraduate Teaching vs. Other Departmental Missions**

- Percentage of faculty loads devoted to undergraduate program versus service, graduate, or research efforts of the department. (IA1.1.1)

- Percentage of faculty reporting that undergraduate program instruction, service course instruction, grant work, disciplinary research is their principal emphasis as a faculty member. (IA1.1.2)

- Percentage emphasis on undergraduate teaching included in department's promotion/tenure criteria and in annual report of allocations of resources. (IA1.1.3)

**Tr: Transactions**

**Departmental priorities and policies concerning undergraduate instruction**

- Proportion of faculty reporting they participate in committees/activities directly tied to the undergraduate program goals and missions. (IA1.2.1)

**Ou: Outcomes**

**Sample Indicator: Departmental Commitment to the Undergraduate Program**

- Proportion of senior faculty involved in direct instruction in the undergraduate program (IA1.3.1)

- Ratio of senior faculty to teaching assistants in terms of credit hours produced. (IA1.3.2)
### Level II: Curriculum

#### An: Antecedents

**Sample Indicator: Expectations for student performance in mathematics**

Full text of item is given in Section B. Curriculum.

- Student performance expectations for departmental examinations (IB1.1.1)
  - Ability to perform routine mathematical procedures (those which are primarily algorithmic and with limited contingent behaviors).
  - Ability to understand or create an appropriate mathematical model (representation) of an everyday situation and to express questions from that situation terms of the model.
  - Ability to provide plausible and mathematically-based justification for a problem solution strategy, conjecture, etc.

#### Tr: Transactions

**Sample Indicator: Changes in curricular delivery**

Indicate with a √ which of the following emphases and/or activities are present in the department's courses and programs (QB1.2.1):

- Use of writing
- Emphasis on modeling
- Emphasis on multiple representations of concepts (graphical, symbolic, tabular, etc.)
- Use of small groups
- Use of extended projects
- Use of alternative modes of assessing student learning (portfolios of work, projects, written reports, "tended open-ended problems"

#### Ou: Outcomes

**Sample Indicator: Ratings of representatives from partner disciplines (e.g. department chairs) of the mathematics department's calculus sequence (IB.1.3.1)**

(for full text of item, see Section D. Classroom)

<table>
<thead>
<tr>
<th>Partner discipline</th>
<th>Appropriate content</th>
<th>Meets student needs</th>
<th>Instruction is effective by students</th>
<th>Perceived as helpful</th>
</tr>
</thead>
<tbody>
<tr>
<td>(illustrative only)</td>
<td>(for the disciplines)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Engineering</td>
<td></td>
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**Level III: Faculty (instructional staff)**

**An: Antecedents**

**Sample Indicator: Specialty, academic background and age distributions of instructional staff (IC1.1)**
(for full text of item, see Section 3. Faculty)

- Number and proportion of faculty in each disciplinary subarea, (e.g., algebra, analysis, statistics) (IC1.1.1)
- Number and proportion of faculty in each 5 year age range, by level of academic preparation. (IC1.1.2)
- Proportion of faculty in each 5 year age range broken out by
  - disciplinary specialties
  - gender
  - rank
  - expertise with technology. (IC1.1.3)

**Tr: Transactions**

**Sample Indicator: Faculty interest/involvement in the undergraduate program**

Note: These data should be reported by faculty rank: years in rank; full or part-time status.

- Proportion of the faculty involved in significant extra-class activities associated with program design and monitoring (IC2.1)
- Proportion of the faculty involved in significant extra-class activities associated with student activities such as math club, Putnam team, actuarial exam preparation (IC2.2)
- Proportion of the faculty involved with undergraduates in significant program or other advisement/counseling activities (IC2.3)
- Proportion of the faculty involved in extra-class student activities (IC2.4)

**Ou: Outcomes**

**Sample Indicator: Faculty development activities in the past three years (IC3)**

To what extent were the following available (mark all that apply):

- Workshops on single topics (example: Using graphing calculators)
- Multiple session, in-depth 'courses' on effective instruction
- Orientation sessions for all new instructional staff on effective teaching
- Arrangements in the department for mentoring concerning teaching performance
- Provision for observation of classroom teaching by peers or by outside specialists
- Periodic seminars or discussion groups on current trends and issues in the teaching and learning of undergraduate mathematics
- Formal training in teaching provided to teaching assistants as a part of their academic program
- Physical support (office space, telephone access, e-mail, etc.) provided to part time faculty and TAs to help them discharge their instructional role.
Level IV: Classroom

An: Antecedents

Sample Indicator: Competencies that are expected of students in the undergraduate program (ID1.2.1)
(For full text of item, see Section D. Classroom)
For each mathematics course, identify the following features of its knowledge base:
__ Major facts, concepts and principles the students are expected to learn;
__ Major procedures and techniques the students are expected to know, and the kinds
   of computations (with and without technology) they should be able to
   perform;
__ Kinds of representations that students are expected to be able to employ
   (e.g. graphs, tables, matrices, sketches )
__ Kinds of connections, within and outside of mathematics, that students are expected
   to be able to make;
__ Ability to communicate (e.g. read, write, speak, listen and model) mathematically.

Tr: Transactions

Sample indicator: Active participation in class work
(Data for this indicator could be obtained from a survey of mathematics department administrators.
__ Proportion of class sections using small group activities at least once per week
   (ID1.1.1)
__ Proportion of students regularly (at least once in a class period) interacting with each
   other about mathematics
   __ in class generally (ID1.1.2)
   __ when given the opportunity to do so in class (D)1.1.3)
   __ in laboratory or discussion sections (ID1.1.4)

Ou: Outcomes

Sample Indicator: (Assessment) A freshman calculus project (ID2.2.1)
(for full text of item, see Section D. Classroom)
The Kingdome has a hydronic heating system which includes three boilers that produce
together 16,500,000 BTUs per hour. It takes 0.0345 BTUs per cubic foot to raise the
temperature one degree.

On Saturday, February 17th at 11:00 am the Kingdome doors will open for the annual
Seattle Home Show....part of your task as a consultant is to determine when the
maintenance crew should turn this heating system on in order to bring the temperature
from a predicted forty-five degrees to seventy-one degrees (the standard temperature
for such an event). The Kingdome must be up to temperature in time for the I I am
opening on the seventeenth.

Problem courtesy of Betty Hawkins, Shoreline Community College, Seattle Washington.
Level V: Students

An: Antecedents

Sample indicator: Background data on students enrolled in mathematics courses
(For full text of item, see Section E. Students)

Student demographics
(IE1.1)
- percentage of students that are black or Hispanic
- percentage residing within 20 miles of campus

Student enrollment behavior (IE1.2)
- percentage of students attending part-time
- percentage of first-time full-time students completing programs
- average time to degree completion

Tr: Transactions

Sample Indicator: Active 'student centered' policies, values and behaviors
- Emphasis placed on contact with students outside of class by individual faculty members (obtained data from survey or interviews (IE2.1)
- Observed frequency of contact between faculty and student immediately before or after class (IE2.2)
- Presence of opportunities for faculty/student contact in the form of study groups, informal study sessions, mathematics clubs, etc. (IE2.3)
- Overall student feelings about faculty contact and accessibility as reported by survey or interviews. (IE2.4)

Ou: Outcomes

Sample Indicator: Identifying student needs and difficulties
- Presence of, and use of results derived from, student satisfaction surveys or reaction questionnaires (IE4.2.1)
- Extent to which student responses on surveys (or in interviews) indicate feeling that their needs and opinions are listened to and taken into account by departmental personnel (M4.2.2)
- Extent to which data (provided, for example, by surveys and interviews) from graduates indicate that the department's curriculum is achieving its goals in terms of student learning and student habits of mind and work in the mathematical sciences (IE4.2.3)
IV. Summary

The development of indicators that give robust descriptions of the health and direction of a department's undergraduate program is a timely topic. Mathematics departments are under pressure both to provide large numbers of service functions as well as to review and consider modifying the direction and emphases of their programs. Insightful, high quality information on past and present program performance is of the essence.

**Methods of Indicator Data Collection**

For many mathematics departments that are considering developing a set of quality indicators for their undergraduate programs, a good place to begin could be with all or part of the Starter Set provided here. In planning to develop such indicators, a department should consider a variety of approaches to collecting the desired data. No one approach will be universally suitable, or even feasible, for collecting the needed information. Thus, it is important to engage in the process persons with diverse perspectives to develop, select and weave together the most appropriate set of indicators. We review below a number of methods which suggest how a department might begin.

The most basic approach for developing a systematic data file on a department's programs is by collecting cross-sectional data. The cross-sectional approach is used most often in developing traditional indicators that provide information for various points in the educational 'pipeline'. Cross-sectional data present current levels, percentages, or some other status measures at regular, or semi-regular, intervals of time. It is important to note that the information is gathered at a group level (class, course, cohort, age cohort, etc), not at the individual level (student, faculty, etc.). In obtaining this information, a major concern is to identify equivalent groups to help ensure that the data are equivalent and enable meaningful comparisons (say, across time).

Cross-sectional data collection is often used in settings where one wishes to identify a trend in a student body or financial/participation factor over time—for example, to monitor the percentage of students in a department's programs who are
participating in mathematics-oriented extracurricular activities. The United States Department of Education's Integrated Post-secondary Education Data System (IPEDS) collects cross-sectional data on a number of aspects of undergraduate programs in mathematics. One such indicator is the number of degrees awarded in mathematics-related fields at particular points in time. Trends in faculty status (full time vs. part time, etc.) and beliefs may be viewed in a cross-sectional context through responses from different faculty cohorts in the Department of Education's National Survey of Post-secondary Faculty (NSOPF).

By contrast, a longitudinal approach to collecting indicator data is better suited for understanding relationships that may exist between system goals, transactions, and outcomes in terms of inputs, processes, and outputs (for example, to evaluate the cumulative effects of an activity over time). Through periodically collecting data from the same students, faculty, or institutions, it becomes possible to develop analytical models that identify how different transactions or experiences affect the status of a department's program. Such data provide a basis for monitoring aspects of a curriculum and the ways in which it is delivered to students. Many postsecondary institutions are now facing state-mandated "value-added assessment" analyses that are related to the students in their programs. Such questions are most likely answered best through longitudinal data.

The federal government develops, or has developed, a number of longitudinal data sets. Foremost among these are the National Educational Longitudinal Study (NELS:88) and the Longitudinal Study of American Youth (LSAY) which provide information on the impact of mathematics course-taking on students' later decisions about course-taking, college enrollment and career goals, as well as data on subsequent earning power.

Longitudinal and cross-sectional data collection designs are not mutually exclusive. They may be combined to provide richer analyses, although such a combination demands more resources. By maintaining an on-going series of cross-sectional data collection activities it is possible with only some additional effort on any desired topic to follow-up on a given group of students or program features to establish
a related longitudinal data set. This is often done in following individuals at given
points in time and across time on the same or different issues.

When first considering the development of indicators, it is natural to consider
quantitative information -- that is, information that can be counted or measured. An
alternative to collecting quantitative indicator data is to use qualitative methodologies--
that is, approaches that involve non-numerical data such as information from classroom
observations, interviews, and case-studies. Such qualitative techniques are especially
well suited for supplementing quantitative approaches, as for example, when a
department is looking for ways to portray the success of its recent graduates or the
'climate' of its classrooms. Case studies of representative students or classes often tell far
more than can individual quantitative indicators.

Qualitative measures of program quality have other merits, as well. They often
provide the reader of reports with a context that helps interpret the numerical data that
are provided. Qualitative data are often useful in developing and validating new,
untested quantitative indicators. They are also excellent for establishing the
relationships between student and faculty attitudes and beliefs and the subsequent
actions or choices that may result.

Quantitative indicators, unless embedded in a well-designed and thoughtful
indicator set, run the risk of providing de-contextualized information. Such information
may seem to have the attributes of reliable, objective data. However, without a
meaningful context, those data may, in fact, not lead to valid judgments and may not be
reliable bases for decision-making even when the numbers generated are themselves
reliable. Indicator data must be both reliable and informative.

**Target Levels for Systems of Indicators**

**Departmental.** In addition to considering a variety of methodological approaches
in developing indicators and collecting information, departments may face related
questions of determining the most appropriate levels for targeting their indicators. At
what level should data be collected and analyzed--individual, classroom, course,
department? Many of the indicators described in this report focus on student experiences and perceptions. This demands data that usually cannot be retrieved from institutional records or data bases. In designing data collections for indicators, departments must consider the level, or "sieve" size. Does the department need data at the global level of freshman/sophomore classes? At the level of liberal arts service programs and major programs? At the level of non-education service courses, mathematics education support courses, or mathematics major courses?

In designing methods of collecting data, a department must consider the questions it wishes to answer both now and in the future. It should also consider what questions it may be called on to answer at some future point in time by a dean or provost's office or by the state's higher education office. Data which can be used flexibly, summed up (aggregated) into broad patterns, or dis-aggregated into more contextualized and specific forms, offer a powerful explanatory tool that is useful for both external (accountability) and internal (program monitoring and planning) purposes. A department should work towards building an optimal indicator set - one that allows addressing individual, department, system, or political questions about the department and its programs in valid, reliable, and defensible ways.

At the same time, the department must consider how such data can be collected, stored, and accessed with minimal time and trouble. Data collected but never analyzed are a waste of time and effort for the faculty, students, and staff involved. However, lacking key data during a critical period of financial or personnel decision making is potentially even more wasteful. The unavailability of appropriate data may increase the likelihood of misusing existing resources or not receiving needed resources. Obviously there is a need for a well considered balance between too much and not enough data.

**National or Federal.** Undergraduate education in mathematics is an empowering stage for almost all post-secondary students, regardless of their level of schooling or their field of study. The effective development of each individual's quantitative competence is an important national concern - a concern for developing a population ready to address quantitative questions in their careers, personal lives, and their lives as fully functioning members of today's society. Concern over the comparable performance
of our students in mathematics is highlighted by public discussion resulting from the release of data from the periodic international mathematics achievement studies or the National Assessments of Educational Progress. Public sector institutions are under increasingly stringent accountability demands while independent colleges are facing the need for more proactive accrediting requirements. There are also increased constraints on fiscal resources—a condition not likely to be reversed in the coming decade. The resulting demands for "consumer responsiveness" and greater efficiencies in delivering instruction at all levels will significantly affect all programs in coming years.

Carefully constructed indicator sets that help to exhibit the productivity of the nation's undergraduate programs, not only in terms of outcomes (number of majors, Ph.D.'s, etc.) but also in terms of student quality and participation, faculty preparation and participation, and the reaction of programs to perceived national needs, can serve as a basis for governmental decision-making regarding funding, program initiation and termination, and new initiatives affecting other levels of schooling. Reliable, informative and substantive indicators also aid in predicting trends and outlining current program needs.

Furthermore, carefully constructed indicator systems, such as those dealing with the topics in the Starter Set provided in this report and extensions of that set, provide a basis for interpreting and understanding changes over time in undergraduate mathematics teaching and learning - changes that can help bring us, as a nation or as a department, closer to the goal of quality mathematics education for all of our students.