Interesting Induction: Student Instruction Guide

Directions: For this lesson, you will need the visual basic (.exe) file and a copy of the activity worksheet. Go to the website http://www.mste.uiuc.edu/courses/ci436fa08/folders/dpikler2/finalproject/ and download the visual basic file contained on the website. The worksheet will be handed to you in class. You MUST be using a PC, as exe files do not open up on macs.

Background Information: Visual Basic is an action orientated programming language, meaning that whenever you click on a field (i.e. button, picture, or a label), press a key, change a field, or even move your mouse over a field, it can execute a set of commands. In fact, you can complete several actions at once, such as check the keys pressed in a textbox as the text is changed, or have the program automatically click a command button when a key is pressed. For those of you that have never programmed in visual basic, don’t worry, as this activity is not about programming, but more about mathematical induction (the visual basic file is pre-made). Double click on the exe to start the activity.

Shortcuts (optional to read, but might be helpful):

- On the command buttons, you might see a letter underlined. This means that if you press alt and the letter at the same time, it will execute the command button without you having to clicking on it.
- There are a few secret key commands to skip pages, and/or go on to the next page. This was done so that you won’t click “Next” the whole way through. Wait for the teacher’s instructions before you go on!!!
- The teacher has the ability to skip to any page of the program at any time. If you feel you are really behind, or accidentally exit the program, call the teacher over so he/she can move you to the right spot.
- Note: The control box typically displayed the top right-hand corner of the screen (allowing you to minimize, resize, and quit the program) were removed because they sometimes interfere with the program. There is an exit button on almost every page if you need to exit the program for some reason, and a secret key combination to exit the review game before it is completed.

Page 1: Individual Work

After you open up the visual basic file, you will see the following screen:
In this activity, we will be looking at the sum of \( n \) cubes, where \( n \) is a number between 1 and 10 (inclusive). For example, \( 1 = 1^3 \), \( 9 = 1^3 + 2^3 \), and so on. Visual basic will do all these calculations. Here is how:

- **Click** on the text box (the only field that is pure white)
- **Type** in the number 1
- **Click** on the button “Test our values” (or press the enter button on the keyboard).

In the box under “Total”, you will see the number “1” displayed and in the “Sum of Cubes” box, you will also see the number “1”. This is because \( 1 = 1^3 \). **Record** this result in table 1 on the student activity worksheet.

If you **repeat** this for the number “2”, you should see two results in the boxes, mainly the results for 1 and 2. Here is what your screen should look like:
Now, repeat this process for the numbers 3 through 10 (in this order). Your sums of cubes should begin to look like a right triangle as the values get higher. Don’t forget to record each of the values on the student activity worksheet.

- If you make a mistake somewhere along the way, you can clear your previous entries and start over. This is the command button to the left of “Test Our Values”.

When you arrive at the number 10, you will see that several actions are toggled. Firstly, you are asked if you notice a pattern in the totals box, and secondly, the “Next” button is now enabled. Write down the pattern you see on the student worksheet (if you see one). Please do NOT click the next button until your classmates are finished.

Page 2: Individual Work

This next page is devoted to looking at squares of sums. It is designed exactly the same as the first page, except when the values 1-10 are entered, it will compute the square of sums instead of the sum of cubes. Just like on the last page, you should do the following:

- Type the numbers 1-10 into the textbox (and click on “Test our values”)
- Record the numbers on the student activity worksheet in table 2.

As you are typing in the numbers 1-10, answer the following question on the student worksheet: Do you see any patterns developing with the sum of squares?

When you arrive at the number 10, another action is toggled and you now can see a button which allows you to view the previous page (called “See Previous Page”). Click on that button to compare your numbers on the second page with the ones on the first page. Although you already have the numbers written down, you can look at them side-by-side on the computer.

What do you notice about the sums (while comparing the values of table 1 to the ones in table 2)? Write your answer down on the activity worksheet. When you are done comparing your values, click on the close button at the bottom right corner of page 1. The “Next” button on page two should now be enabled, allowing you to move on. Again, wait for your classmates to finish before clicking “Next”.

Page 3: Group Work

This page is to be done together as a group. When you reach this point, you should notice that the numbers on the first page are the same as the numbers on the second page. If you did not arrive at this conclusion, call your teacher over now. Here is your task:
• Write one down an equation relating the sum of cubes to the square of sums on the activity worksheet. Use summation notation to be more precise, utilizing the variable \( n \) as your \( n \)th term.
• The teacher will call on someone to present their answer at the smartboard. Until this happens, sit tight and relax, because you will be unable to go any further. Please wait for the teacher’s instructions!!!

After the teacher has told you how to move on, a message box will appear asking if you want to see the correct equation. Click on “Yes”. The summation relationship is displayed, and the “Next” button should now be enabled.

Click on the “Next” button. We are now asked to attempt to show this equation is true via induction. Even though we showed it is true for the first 10 values, we have not proved it to be true for all \( n \), so we must use induction to show this fact always true. Your screen (minus the relationship) should now look like this:

We will work on this step together, so the “Next page” button has been disabled. As a heads up, the induction proof will be displayed on the next page. After we work through this proof, wait for the teacher to give out the secret keystroke to enable the “Next page” command button so you can see the proof written out on your computer.

**Page 4: Group Work**

This page consists of the induction proof of the sum of cubes/square of sums problem. It is recommended that you follow each of the steps to better understand how induction works. Write down each of the steps if you did not get them copied down as we went through it in class. It is organized in three steps, including a basis step, inductive assumption, and then a formal proof showing that this relationship is true for the \((n+1)\)st term. Click the “Next Step” button to show each of the three steps, and then click on the “Next page” button. Since we have worked this out as a group, I only expect that you
look over the proof one more time for extra practice. Be sure to the teacher any questions that you have.

**Page 5: Individual Work/Group Work**

This page should look very similar to page 1 and 2. Just like before, here is what you need to do:

- **Click** on the text box (the only field that is pure white)
- **Type** in the numbers 1-10
- **Click** on the button “Test our values” for each number.

**Record** your results in table 3 on your activity worksheet. Then develop a pattern for these numbers in summation form just like you did in the last problem. Here is a picture of what your screen should look like after you type in the values 1 and 2:

![Pattern Development Screen](image)

**Page 6: Group Work**

This page will consist of an example of how to compute the nth term of the sum of even numbers. There are four steps we will use for this computation, which we will go through as a group. Follow the teacher as each step is shown (clicking the “Next Step” button at the appropriate times). Be sure to **write down** these four steps on your activity worksheet!!! It will be useful for the next page (final review activity), since you will have to use these four steps in order to complete the review activity. **Click** on the Next button once we have finished discussing the problem.

**Page 7: Individual/Partner Activity**

After **clicking** on the “Next” button, you will get a message box telling you that there is a review game to practice finding the nth term of a series. **Clicking** “ok” will make the following screen appear:
Directions for the Review Activity: This activity can be done with a partner, or individually. The goal is to match the summation with the nth term form (we can discover the nth term form by using the four steps in the last problem). Some of these relationships will be easier to see than others. The computer will tally your number of wrong guesses, and will display them when you finish the activity. The goal is to get as many correct in with the least number of wrong guesses.

There is a key code to reset the entire board, undo a move, and also to exit the game prematurely, so call the teacher over if you wish to do one of these actions. When you finish the review game, record the number of wrong guesses on your activity worksheet.

<table>
<thead>
<tr>
<th>Summation Formulation</th>
<th>nth Term Formulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sum_{i=1}^{n} x)</td>
<td>(2(2^x - 1))</td>
</tr>
<tr>
<td>(\frac{n}{n+1})</td>
<td>(\sum_{i=1}^{n} \left(\frac{1}{i}\right))</td>
</tr>
<tr>
<td>(\sum_{i=1}^{n} \frac{1}{(3a-2)(3a+1)})</td>
<td>(\sum_{i=1}^{n} \frac{1}{k(k+1)})</td>
</tr>
<tr>
<td>(\sum_{i=1}^{n} i^2)</td>
<td>(\sum_{i=1}^{n} 2^x)</td>
</tr>
</tbody>
</table>

\(n\) \(n(n+1)\) \(n^2\)